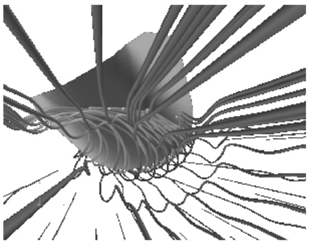

9èmes JOURNÉES DE L'HYDRODYNAMIQUE

10 - 11 - 12 MARS 2003

POITIERS - FUTUROSCOPE

WEAKLY-SINGULAR BOUNDARY-INTEGRAL REPRESENTATIONS FOR WAVE DIFFRACTION-RADIATION WITH FORWARD SPEED

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Résumé

Cette étude porte sur le problème fondamental qui consiste à déterminer l'écoulement potentiel à surface libre correspondant à un écoulement donné sur la coque d'un navire ou d'une structure offshore. On s'intéresse tout particulièrement à la diffraction-radiation de vagues harmoniques, avec ou sans vitesse d'avance, ainsi qu'aux écoulements permanents et aux cas limites correspondant à une gravité nulle ou infinie. Des représentations intégrales du potentiel qui ne font intervenir que des fonctions qui ne sont pas plus singulières qu'une fonction de Green G sont obtenues. Ces représentations intégrales sont faiblement singulières par rapport aux représentations classiques du potentiel, qui font intervenir G et ∇G. Les applications numériques présentées dans *Noblesse et al. (2002)*, pour un écoulement permanent, montrent que les annulations numériques que l'on observe entre l'intégrale de surface sur la coque du navire et l'intégrale de ligne le long de la ligne d'eau lorsqu'on utilise la représentation classique du potentiel due à *Brard (1972)* disparaissent en grande partie si la représentation faiblement singulière du potentiel est utilisée.

Summary

The fundamental problem of determining the free-surface potential flow that corresponds to a given flow at the wetted surface of a ship or offshore structure is considered. Diffraction-radiation of time-harmonic waves with forward speed, and the special cases corresponding to diffraction-radiation without forward speed, steady flow, and the infinite-gravity and zero-gravity limits, are examined. Boundary-integral representations that define the velocity potential ϕ in terms of functions that are no more singular than a Green function G are given. These boundary-integral representations are weakly singular in comparison to the classical boundary-integral representations of free-surface potential flows, which define the potential ϕ in terms of G and ∇G. The illustrative numerical applications presented in *Noblesse et al. (2002)* show that the numerical cancellations that occur between the waterline integral and the hull-surface integral in the classical boundary-integral representation of steady ship waves are largely eliminated in the corresponding weakly-singular boundary-integral representation.

1. Introduction

Theoretical prediction of the behavior of ships and offshore structures in time-harmonic ambient waves is one of the most important core issues in free-surface hydrodynamics. For offshore structures, robust and highly-efficient panel methods have been developed, and are routinely used, to solve the basic wave radiation-diffraction problems required to predict added-mass and wave-damping coefficients, and wave-exciting forces and moments. These potential-flow methods are based on numerical solution of a boundary-integral equation associated with the Green function corresponding to the linear free-surface boundary condition for diffraction-radiation of time-harmonic waves without forward speed. Application of this classical approach, often identified as the free-surface Green function method, to wave diffraction-radiation by ships (i.e. with forward speed) has also led to useful methods — see e.g. *Boin et al. (2002,2000), Ba et al. (2001), Chen et al. (2000), Guilbaud et al. (2000), Fang (2000), Wang et al. (1999), Du et al. (2000,1999), Zhang and Eatock-Taylor (1999), Iwashita and Ito (1998), Iwashita (1997)* — although not to a comparable degree of practicality because forward speed introduces major difficulties (not present for wave diffraction-radiation at zero forward speed).

A fundamental difficulty, mathematical in nature, is related to the fact that the boundary-integral representation of time-harmonic (or steady) free-surface flows about ships (advancing with forward speed $\mathcal{U}$) involves a line integral along the ship waterline. This line integral (not present if $\mathcal{U}=0$) has been shown to have important effects, see e.g. *Ba et al. (2001)* and *Zhang and Eatock-Taylor (1999).* In particular, the contribution of the waterline integral is shown in *Ba et al. (2001)* to have a large effect upon the occurrence of irregular frequencies. In addition, the contribution of the waterline integral largely cancels out the contribution of the surface integral over the ship hull. These numerical cancellations, numerically illustrated in *Noblesse and Yang (1995)* and *Noblesse et al. (2002)*, can result in serious losses of accuracy. The numerical/mathematical difficulty associated with the waterline integral that occurs if $\mathcal{U}\neq 0$ is addressed in the present study, which reconsiders the fundamental problem of determining the free-surface potential flow that corresponds to a given flow at the wetted hull surface of a ship (or offshore structure).

A classical boundary-integral representation, based on one of Green's fundamental identities, defines the velocity potential at a field point inside a flow domain in terms of the potential ϕ and its normal derivative $\partial\phi/\partial n$ at the boundary surface. As already noted, the corresponding classical boundary-integral representation of potential flow due to a ship advancing at constant speed in time-harmonic waves (or in calm water) involves a line integral along the ship waterline; see e.g. *Brard (1972)*. The waterline integral in this classical boundary-integral representation of time-harmonic ship waves stems from a transformation, based on Stokes' theorem, of the integral over the mean free-surface plane in Green's identity. Both the line integral along the ship waterline and the surface integral over the ship wetted hull involve first derivatives of the Green function G associated with the Michell free-surface boundary condition. Specifically, the waterline integral involves the derivative G_x of G along the path of the ship, and the hull integral involves ∇G. Numerical cancellations related to these complicated and highly-singular functions can result in serious losses of accuracy, as already noted.

Here, an integration by parts is performed to transform the distribution of normal dipoles over the boundary surface in the classical boundary-integral representation of the potential. This transformation yields boundary-integral representations of potential flows that define the potential in terms of a Green function G and related functions that are no more singular than G. Thus, the boundary-integral representations of the potential given in this study are weakly singular in comparison to the corresponding classical boundary-integral representations, which define the potential ϕ in terms of G and ∇G. The illustrative numerical applications presented in *Noblesse et al. (2002)* show that the numerical cancellations that occur between the waterline integral and the hull integral in the classical boundary-integral representation of steady ship waves are largely eliminated in the corresponding weakly-singular boundary-integral representation, which may then be useful for numerical purposes.

2. Problem statement

Consider potential flow about a ship or other floating rigid body, e.g. an offshore structure, at or below the free surface of a large body of water of uniform depth D. Let Σ_B be a surface located outside the viscous boundary layer that surrounds the ship hull. The surface Σ_B includes the outer edge of the viscous wake trailing the ship, or a surface outside the viscous wake. If viscous effects are ignored, Σ_B may be taken as the mean wetted ship hull. For a ship equipped with lifting surfaces, e.g. a sailboat, Σ_B also includes the two sides of every vortex sheet behind the ship hull. For a multihull ship, the hull+wake surface Σ_B consists of several component surfaces, which correspond to the separate hull components of the ship and their wakes.

The flow domain is bounded by the surface

$$\Sigma = \Sigma_B \cup \Sigma_0 \cup \Sigma_D$$

where Σ_0 is the portion of the mean free-surface plane $Z=0$ located outside the "body" surface Σ_B and Σ_D is the sea floor $Z=-D$. Let Γ represent the intersection curve between the surfaces Σ_B and Σ_0, i.e. the intersection curve of the body surface Σ_B with the free-surface plane. The unit vector $\vec{n} = (n^x, n^y, n^z)$ is normal to the boundary surface Σ and points into the flow domain. Thus, $\vec{n} = (0, 0, -1)$ at the free surface Σ_0 and $\vec{n} = (0, 0, 1)$ at the sea floor Σ_D. The unit vector $\vec{t} = (t^x, t^y, 0)$ is tangent to the boundary curve Γ and oriented clockwise (looking down).

Coordinates are nondimensional with respect to a reference length L, e.g. the ship length. The nondimensional water depth is $d=D/L$. The fluid velocity is nondimensional with respect to a characteristic reference velocity U, e.g. the ship speed $\mathcal{U}$, and the velocity potential is nondimensional with respect to the reference potential UL. The z axis is vertical and points upward, and the mean free surface is taken as the plane $z=0$ as already noted. For steady and time-harmonic flow about a ship advancing in calm water or in waves, the x axis is chosen along the path of the ship and points toward the ship bow.

Let $\vec{\xi} = (\xi, \eta, \zeta)$ and $\vec{x} = (x, y, z)$ stand for a field point and a singularity point, respectively, associated with a Green function $G(\vec{x}\,;\vec{\xi})$. The field point $\vec{\xi}$ lies inside the flow domain, and the singularity point $\vec{x}$ is located on the boundary surface Σ. Hereafter, ϕ_* stands for the velocity potential at a field point $\vec{\xi}$, and ϕ and $\vec{u} = \nabla\phi$ represent the potential and the velocity at a boundary point $\vec{x}$. Furthermore, ∇ and ∇_* stand for the differential operators $\nabla = (\partial_x, \partial_y, \partial_z)$ and $\nabla_* = (\partial_\xi, \partial_\eta, \partial_\zeta)$.

The sea floor $z=-d$ is assumed to be a rigid wall. Thus, the sea-floor boundary condition

$$w = 0 \quad \text{at} \quad z = -d \tag{1a}$$

holds. Furthermore, Green functions G that satisfy the sea-floor boundary condition

$$G_z = 0 \quad \text{at} \quad z = -d \tag{1b}$$

are considered here.

Define u^n and $\vec{\Omega} = (\Omega^x,\, \Omega^y,\, \Omega^z)$ as

$$u^n = \vec{u}\cdot\vec{n} \qquad\qquad \vec{\Omega} = \vec{u}\times\vec{n} = \vec{u}^{\,\theta}\times\vec{n} \qquad \text{with} \qquad \vec{u}^{\,\theta} = \vec{u} - u^n\,\vec{n} \tag{2a}$$

Thus, u^n represents the component of the velocity $\vec{u}$ along the unit vector $\vec{n}$ normal to the boundary surface Σ and $\vec{u}^{\,\theta}$ is the tangential component of $\vec{u}$. The expressions

$$\vec{\Omega} = \left\{\begin{array}{c} \Omega^x = n^z v - n^y w \\ \Omega^y = n^x w - n^z u \\ \Omega^z = n^y u - n^x v \end{array}\right\} \qquad\qquad \vec{u}^{\,\theta} = \left\{\begin{array}{c} u - u^n n^x \\ v - u^n n^y \\ w - u^n n^z \end{array}\right\} \tag{2b}$$

yield $\vec{\Omega} = (-v\,, u\,, 0)$ and $\vec{u}^{\,\theta} = (u\,, v\,, 0)$ at the free-surface plane.

3. Classical representation of potential flows

The potential ϕ_* at a field point $\vec{\xi}$ within a flow domain is defined in terms of the boundary values of the potential ϕ and its normal derivative $\nabla\phi\cdot\vec{n} = \vec{u}\cdot\vec{n} = u^n$ by the classical boundary-integral representation

$$\phi_* = \int_\Sigma d\mathcal{A}\,(\,u^n\,G - \phi\;\vec{n}\cdot\nabla G\,) \tag{3}$$

where $d\mathcal{A}$ stands for the differential element of area at a point $\vec{x}$ of the boundary surface Σ. This representation defines the potential in terms of boundary distributions of sources (with strength u^n) and normal dipoles (with strength ϕ), and involves a Green function G and first derivatives of G. The velocity field $\vec{u}_*$ associated with the classical potential representation (3) is given by

$$\vec{u}_* = \nabla_*\phi_* = \int_\Sigma d\mathcal{A}\,[\,u^n\,\nabla_* G - \phi\;\nabla_*\,(\,\vec{n}\cdot\nabla G\,)\,] \tag{4}$$

This representation of the velocity involves second derivatives of G.

The classical boundary-integral representation (3) holds for a field point $\vec{\xi}$ inside the flow domain, strictly outside the boundary surface Σ. This property is related to the well-known fact that the potential defined by the dipole distribution in (3) is not continuous at the surface Σ. Indeed, (3) becomes

$$\frac{1}{2}\,\phi_* = \int_\Sigma d\mathcal{A}\,(\,u^n\,G - \phi\;\vec{n}\cdot\nabla G\,)$$

at a point $\vec{\xi}$ of the boundary surface Σ (if Σ is smooth at $\vec{\xi}$).

4. Weakly-singular representations of potential flows

Consider the vector fields $\vec{g}_1\,, \vec{g}_2\,, \vec{g}_3$ defined as

$$\left\{\begin{array}{l} \vec{g}_1 = \nabla G^x \times \vec{i} = (\,0\,, G^x_z\,, -G^x_y\,) \\ \vec{g}_2 = \nabla G^y \times \vec{j} = (-G^y_z\,, 0\,, G^y_x\,) \\ \vec{g}_3 = \nabla G^z \times \vec{k} = (G^z_y\,, -G^z_x\,, 0\,) \end{array}\right\} \quad \text{where} \quad \left\{\begin{array}{l} \vec{i} = (1\,, 0\,, 0) \\ \vec{j} = (0\,, 1\,, 0) \\ \vec{k} = (0\,, 0\,, 1) \end{array}\right\} \tag{5a}$$

and a subscript/superscript indicates differentiation/integration. These vector fields satisfy the identity

$$\nabla\times\vec{g} = \nabla G \tag{5b}$$

for a function G, e.g. a Green function, that satisfies the Laplace equation. The identity

$$\nabla\times(\,\phi\,\vec{g}\,) = \phi\,\nabla\times\vec{g} + \nabla\phi\times\vec{g}$$

and the relation (5b) yield

$$\nabla\times(\phi\vec{g}) = \phi\nabla G - \vec{g}\times\nabla\phi$$

Furthermore, the relations $\vec{u} = \nabla\phi$ and $\vec{\Omega} = \vec{u}\times\vec{n}$ yield

$$\vec{n}\cdot[\nabla\times(\phi\vec{g})] = \phi\,\vec{n}\cdot\nabla G - \vec{g}\cdot\vec{\Omega} \tag{5c}$$

Expressions (5) relate usual (scalar) Green functions G and corresponding vector Green functions $\vec{g}$.

Integration of (5c) over the boundary surface Σ yields the three alternative transformations

$$\int_\Sigma d\mathcal{A}\;\phi\,\vec{n}\cdot\nabla G = \int_\Sigma d\mathcal{A}\left\{\begin{matrix}\nabla G^x\times\vec{i}\\ \nabla G^y\times\vec{j}\\ \nabla G^z\times\vec{k}\end{matrix}\right\}\cdot\vec{\Omega} = \int_\Sigma d\mathcal{A}\left\{\begin{matrix}\Omega^y G^x_z - \Omega^z G^x_y\\ \Omega^z G^y_x - \Omega^x G^y_z\\ \Omega^x G^z_y - \Omega^y G^z_x\end{matrix}\right\} \tag{6a}$$

The field point $\vec{\xi}$ in (6a) is within the flow domain, strictly outside the boundary surface Σ. The transformations (6a) express a surface integral involving the potential ϕ and the derivative ∇G of a Green function G as an integral that involves $\vec{\Omega} = \nabla\phi\times\vec{n}$ and the functions ∇G^x, ∇G^y or ∇G^z, which are no more singular than G. Thus, the transformations (6a) correspond to an integration by parts $(\phi,\nabla G)\to(\nabla\phi, G)$. Differentiation of the transformations (6a) yields

$$\int_\Sigma d\mathcal{A}\;\phi\,\nabla_*(\vec{n}\cdot\nabla G) = \int_\Sigma d\mathcal{A}\;\nabla_*\left\{\begin{matrix}\nabla G^x\times\vec{i}\\ \nabla G^y\times\vec{j}\\ \nabla G^z\times\vec{k}\end{matrix}\right\}\cdot\vec{\Omega} = \int_\Sigma d\mathcal{A}\;\nabla_*\left\{\begin{matrix}\Omega^y G^x_z - \Omega^z G^x_y\\ \Omega^z G^y_x - \Omega^x G^y_z\\ \Omega^x G^z_y - \Omega^y G^z_x\end{matrix}\right\} \tag{6b}$$

with $\nabla_* = (\partial_\xi, \partial_\eta, \partial_\zeta)$. An interesting special case of the family of three alternative transformations (6b) is obtained by differentiating the first, second and third of these transformations with respect to ξ, η and ζ, respectively, i.e.

$$\int_\Sigma d\mathcal{A}\;\phi\left\{\begin{matrix}\partial_\xi(\vec{n}\cdot\nabla G)\\ \partial_\eta(\vec{n}\cdot\nabla G)\\ \partial_\zeta(\vec{n}\cdot\nabla G)\end{matrix}\right\} = \int_\Sigma d\mathcal{A}\left\{\begin{matrix}\nabla G^x_\xi\times\vec{i}\\ \nabla G^y_\eta\times\vec{j}\\ \nabla G^z_\zeta\times\vec{k}\end{matrix}\right\}\cdot\vec{\Omega} = \int_\Sigma d\mathcal{A}\;\vec{\Omega}\times\nabla_* G \tag{6c}$$

for a Green function that satisfies the identity $\nabla_* G = -\nabla G$.

By substituting the transformation (6c) into the classical representation (4) of the velocity, we obtain

$$\vec{u}_* = \int_\Sigma d\mathcal{A}\,(u^n\,\nabla_* G - \vec{\Omega}\times\nabla_* G) \tag{7}$$

This weakly-singular representation of the velocity is well known; see e.g. *Hunt (1980).* Substitution of the transformation (6a) into the classical representation (3) of the potential yields

$$\phi_* = \int_\Sigma d\mathcal{A}\,(u^n\,G - \left\{\begin{matrix}\Omega^y G^x_z - \Omega^z G^x_y\\ \Omega^z G^y_x - \Omega^x G^y_z\\ \Omega^x G^z_y - \Omega^y G^z_x\end{matrix}\right\}) \tag{8}$$

The representations of the velocity $\nabla_*\phi_*$ associated with the family of three alternative weakly-singular representations (8) of ϕ_* generalize the classical velocity representation (7). The three alternative weakly-singular representations (8) of the potential, and the corresponding weakly-singular representations of the velocity, are applied to steady ship waves in *Noblesse et al.*

(2002). The comparison of the steady farfield ship waves associated with the alternative flow representations (8) given in that study indicates that the potential representation corresponding to the vector field $\vec{g}_3$ is more useful than the alternative representations associated with $\vec{g}_1$ and $\vec{g}_2$. Thus, hereafter we only consider the potential representation associated with $\vec{g}_3$, i.e.

$$\phi_* = \int_\Sigma d\mathcal{A}\,(\,u^n\,G + \Omega^y\,G_x^z - \Omega^x\,G_y^z\,) \tag{9}$$

This boundary-integral representation is now considered in the simplest case when the Green function G is taken as the basic free-space Green function.

5. Free-space Green function

Consider the basic (free-space) Green function given by $4\pi G = -1/r$ with

$$r = \sqrt{\vec{X}\cdot\vec{X}} \qquad\qquad \vec{X} = (\,X, Y, Z\,) = (\,x-\xi\,, y-\eta\,, z-\zeta\,) \tag{10a}$$

The function $(1/r)^z$ and its derivatives with respect to ξ and η are given by

$$(1/r)^z = \text{sign}(Z)\,\ln(\,r + |\,Z\,|\,) \qquad\qquad \begin{Bmatrix} (1/r)_x^z \\ (1/r)_y^z \end{Bmatrix} = \frac{\text{sign}(Z)}{r\,(\,r + |\,Z\,|\,)} \begin{Bmatrix} X \\ Y \end{Bmatrix} \tag{10b}$$

The corresponding vector Green function $\vec{g}$, defined by (5a) as $\vec{g} = \nabla(1/r)^z \times \vec{k}$, can be verified to satisfy the relation $\nabla\times\vec{g} = \nabla(1/r)$ in agreement with (5b).

By using (10b) in (9), we obtain

$$\phi_* = \frac{-1}{4\pi}\int_\Sigma \frac{d\mathcal{A}}{r}\left(u^n + \text{sign}(\delta^z)\,\frac{\Omega^y\,\delta^x - \Omega^x\,\delta^y}{1 + |\,\delta^z\,|}\right) \quad \text{with} \quad \vec{\delta} = \frac{\vec{X}}{r} \tag{11}$$

For purposes of verification, the potential representation (11) can be considered for a simple case. E.g., the potential defined by the representation (11) — where the boundary surface Σ is taken as a sphere and the velocity at Σ is that generated by a dipole at the center of the sphere — can be verified to be identical to the potential of the flow due to the dipole, outside and on the spherical boundary surface Σ.

Consider the representation (11) of the potential $\phi_* = \phi(\vec{\xi})$ for a field point $\vec{\xi}$ located on the flow side of the boundary surface Σ. The surface Σ can be decomposed into a local region Σ^ε surrounding $\vec{\xi}$ and the region $\Sigma - \Sigma^\varepsilon$. If Σ is smooth at the point $\vec{\xi}$, the local region Σ^ε may be taken as a circular disk, with radius ε, centered at $\vec{\xi}$ in the tangent plane to Σ at $\vec{\xi}$. The contribution ϕ_*^ε due to Σ^ε is easily shown to be $O(\varepsilon)$ and thus vanishes in the limit $\varepsilon \to 0$. It follows that the weakly-singular representation (11) defines a potential ϕ_* that is continuous at the boundary surface Σ, unlike the classical boundary-integral representation (3). Specifically, the transformations (6a), where the field point $\vec{\xi}$ is within the flow domain (strictly outside the boundary surface Σ), also hold for a point $\vec{\xi}$ at the surface Σ if the left side of (6a) is replaced by $\int_\Sigma d\mathcal{A}\;\phi\;\vec{n}\cdot\nabla G - \phi_*/2$.

6. Application to free-surface flows

The contribution of the sea floor Σ_D to the alternative potential representations (3) and (9) is null for a Green function that satisfies the condition (1b) at the sea floor, as assumed here. Thus, (3) becomes

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\;\vec{n}\cdot\nabla G) - \int_{\Sigma_0} dx\,dy\,(w\,G - \phi\,G_z) \tag{12a}$$

At the free surface Σ_0, we have $(\Omega^x, \Omega^y) = (-v\,, u)$ in accordance with (2b). Thus, (9) yields

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y\,G_x^z - \Omega^x\,G_y^z) - \int_{\Sigma_0} dx\,dy\,(w\,G - u\,G_x^z - v\,G_y^z) \tag{12b}$$

The classical boundary-integral representation (12a) defines the potential ϕ_* at a field point $\vec{\xi}$ in terms of boundary values of the normal velocity component u^n and the potential ϕ, and involves a Green function G and its gradient ∇G. The alternative representation (12b) defines the potential ϕ_* in terms of boundary values of the velocity $\vec{u}$, and involves a Green function G and the related functions G_x^z and G_y^z, which are no more singular than G. The alternative potential representations (12a) and (12b) are further considered below for diffraction-radiation of time-harmonic waves with forward speed, and the special cases corresponding to diffraction-radiation without forward speed, steady flow, and the infinite-gravity and zero-gravity limits.

7. Flows in infinite-gravity and zero-gravity limits

Free-surface flows in the infinite-gravity and zero-gravity limits, associated with the boundary conditions $w=0$ (infinite-gravity flow) and $\phi=0$ (zero-gravity flow) at the plane $z=0$, are now considered. More generally, the nonhomogeneous problems corresponding to a specified vertical velocity w or potential ϕ at the plane $z=0$ are considered. In the infinite-gravity limit, the Green function G is chosen to satisfy the boundary condition $G_z=0$ (and thus also $G^z=0$ as verified further on) at $z=0$. The classical potential representation (12a) and the weakly-singular representation (12b) then become

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\;\vec{n}\cdot\nabla G\,) - \int_{\Sigma_0} dx\,dy\;w\,G \tag{13a}$$

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y\,G_x^z - \Omega^x\,G_y^z\,) - \int_{\Sigma_0} dx\,dy\;w\,G \tag{13b}$$

respectively. In the zero-gravity limit, the Green function G is chosen to satisfy the boundary condition $G=0$ at $z=0$. The potential representations (12a) and (12b) then become

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\;\vec{n}\cdot\nabla G\,) + \int_{\Sigma_0} dx\,dy\;\phi\,G_z \tag{14a}$$

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y\,G_x^z - \Omega^x\,G_y^z\,) + \int_{\Sigma_0} dx\,dy\,(u\,G_x^z + v\,G_y^z\,) \tag{14b}$$

respectively. The boundary-integral representations (13b) and (14b) define the potential ϕ_* at a field point $\vec{\xi}$ inside the flow domain in terms of the normal and tangential components u^n and $\vec{\Omega}$ of the velocity $\vec{u}$ at the body surface Σ_B and the (presumed known, null in typical cases) normal velocity w (infinite-gravity limit) or tangential velocity components u and v (zero-gravity limit) at the plane $z=0$.

In the deep-water limit $d\to\infty$, the Green function G may be taken as $G = R^-$ for the infinite-gravity limit and $G = R^+$ for the zero-gravity limit, with

$$4\pi \begin{Bmatrix} R^- \\ R^+ \end{Bmatrix} = \begin{Bmatrix} -1/r - 1/r' \\ -1/r + 1/r' \end{Bmatrix} \tag{15}$$

Here, r is given by (10a) and r' is defined as

$$r' = \sqrt{\vec{X}'\cdot\vec{X}'} \qquad\qquad \vec{X}' = (\,X, Y, Z'\,) = (\,x-\xi\,, y-\eta\,, z+\zeta\,) \tag{16a}$$

The function $(1/r')^{\,z}$ and its derivatives with respect to ξ and η are given by

$$(1/r')^{\,z} = -\ln(\,r' - Z'\,) \qquad \begin{Bmatrix} (1/r')^{z}_{x} \\ (1/r')^{z}_{y} \end{Bmatrix} = \frac{-1}{r'\,(\,r' - Z'\,)} \begin{Bmatrix} X \\ Y \end{Bmatrix} \tag{16b}$$

Here, the restrictions $z \le 0$ and $\zeta \le 0$ were used. At the plane $z\!=\!0$, (10) and (16) yield

$$(1/r)^{\,z} = \ln(\,r - \zeta\,) \qquad (1/r')^{\,z} = -\ln(\,r' - \zeta\,)$$

Thus, the Green function R^- satisfies the boundary condition $(R^-)^{\,z}\!=\!0$ at $z\!=\!0$ as previously assumed. Expressions (10b) and (16b) show that the integral over the body surface Σ_B in the representations (13b) and (14b) are given by

$$\frac{-1}{4\pi}\int_{\Sigma_B} d\mathcal{A}\left(u^n\,(\,\frac{1}{r} \pm \frac{1}{r'}\,) + (\,\Omega^y X - \Omega^x\, Y\,)\,(\,\frac{\operatorname{sign}(Z)}{r\,(r + |\,Z\,|\,)} \mp \frac{1}{r'\,(\,r' - Z'\,)}\,)\right) \tag{17}$$

with $X, Y, Z\,, Z', r$ and r' given by (10a) and (16a). The integrand of the surface integral (17) is no more singular than $1/r$.

8. Time-harmonic flows with forward speed

Diffraction-radiation by a ship advancing (at constant speed $\mathcal{U}$) in time-harmonic waves (frequency ω) is now considered. Define the nondimensional wave frequency f, the Froude number F, and $\widehat{\tau}$ as

$$f = \omega\sqrt{L/g} \qquad F = \mathcal{U}/\sqrt{gL} \qquad \widehat{\tau} = 2\,F f \tag{18}$$

Also define Φ and $\mathcal{G}$ as

$$\Phi = w - f^2\phi + i\,\widehat{\tau}\,u + F^2 u_x \tag{19a}$$
$$\mathcal{G} = G_z - f^2 G - i\,\widehat{\tau}\,G_x + F^2 G_{xx} \tag{19b}$$

The integrand of the integral over the free surface Σ_0 in (12a) can be expressed as

$$w\,G - \phi\,G_z = \Phi\,G - \phi\,\mathcal{G} - i\,\widehat{\tau}(\phi\,G)_x - F^2(u\,G - \phi\,G_x)_x$$

By using this identity and Stokes' theorem in the integral over the free surface Σ_0 in the boundary-integral representation (12a), we obtain

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\,\vec{n}\cdot\nabla G) - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - \phi\,\mathcal{G}) + \int_{\Gamma} d\mathcal{L}\,[\,i\,\widehat{\tau}\,\phi\,G + F^2(u\,G - \phi\,G_x)\,]\,t^y \tag{20a}$$

The integrand of the integral over the free surface Σ_0 in (12b) can be expressed as

$$\begin{aligned} w\,G - u\,G^z_x - v\,G^z_y = {} & \Phi\,G - \,u\,\mathcal{G}^{zz}_x - v\,\mathcal{G}^{zz}_y - f^2\,[\,(\phi\,G^{zz}_x)_x + (\phi\,G^{zz}_y)_y\,] \\ & - i\,\widehat{\tau}\,[\,(v\,G^{zz}_y)_x - (u\,G^{zz}_y)_y\,] - F^2\,[\,(u\,G)_x - (v\,G^{zz}_{xy})_x + (u\,G^{zz}_{xy})_y\,] \end{aligned}$$

By using this identity, Stokes' theorem, and the identity $\vec{u}\cdot\vec{t} = \phi_t$ in (12b), we obtain

$$\begin{aligned} \phi_* = {} & \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y G^z_x - \Omega^x G^z_y) - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - u\,\mathcal{G}^{zz}_x - v\,\mathcal{G}^{zz}_y) \\ & + \int_{\Gamma} d\mathcal{L}\,[\,f^2\phi\,(t^y G^{zz}_x - t^x G^{zz}_y) + i\,\widehat{\tau}\,\phi_t\,G^{zz}_y + F^2(t^y u\,G - \phi_t\,G^{zz}_{xy})\,] \end{aligned} \tag{20b}$$

The integral over the body surface Σ_B in the classical potential representation (20a) involves the derivative ∇G of G, and the integral along the boundary curve Γ involves the derivative G_x. The integrals over the boundary surface Σ_B and along the boundary curve Γ in the alternative representation (20b) involve the Green function G and the related functions G_x^z, G_y^z, G_{xy}^{zz}, G_x^{zz} and G_y^{zz}, which are no more singular than G.

The integrals over the free surface Σ_0 in (20a) and (20b) are null if the potential is assumed to satisfy the Michell linear free-surface boundary condition $\Phi = 0$ associated with diffraction-radiation of time-harmonic waves by a ship, and if a Green function that satisfies the related free-surface boundary condition $\mathcal{G} = 0$ is used. The function Φ is not null for a surface-effect ship involving a pressure distribution over the free surface, or if nearfield effects are taken into account. In any case, the linearized free-surface boundary condition $\Phi = 0$ holds in the farfield, and integration of Φ is required only over a finite nearfield region of the free surface Σ_0 in the vicinity of the boundary curve Γ.

If $F = 0$, i.e. for the special case of time-harmonic flows at zero forward speed, the classical representation (20a) becomes

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\,\vec{n}\cdot\nabla G) - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - \phi\,\mathcal{G}) \tag{21a}$$

with $\Phi = w - f^2\phi$ and $\mathcal{G} = G_z - f^2 G$, and the representation (20b) can be expressed as

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y G_x^z - \Omega^x G_y^z) + \int_{\Gamma} d\mathcal{L}\,(t^y G_x^z - t^x G_y^z)\,\phi - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - \phi\,\mathcal{G}) \tag{21b}$$

A notable property of the classical potential representation (21a) is that it does not involve a line integral along the boundary curve Γ, unlike the weakly-singular representation (21b). If $f = 0$, i.e. for steady flows, the potential representations (20a) and (20b) become

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G - \phi\,\vec{n}\cdot\nabla G) + F^2\int_{\Gamma} d\mathcal{L}\;t^y(u\,G - \phi\,G_x) - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - \phi\,\mathcal{G}) \tag{22a}$$

$$\phi_* = \int_{\Sigma_B} d\mathcal{A}\,(u^n\,G + \Omega^y G_x^z - \Omega^x G_y^z) + F^2\int_{\Gamma} d\mathcal{L}\,(t^y u\,G - \phi_t\,G_{xy}^{zz}) - \int_{\Sigma_0} dx\,dy\,(\Phi\,G - u\,\mathcal{G}_x^{zz} - v\,\mathcal{G}_y^{zz}) \tag{22b}$$

with $\Phi = w + F^2 u_x$ and $\mathcal{G} = G_z + F^2 G_{xx}$. The representation (22a) is given in *Brard (1972).*

9. Comparison of alternative potential representations

The alternative potential representations (20a) and (20b) are now compared in the case when the Green function G satisfies the free-surface boundary condition $\mathcal{G} = 0$. This Green function can be expressed in terms of simple Rankine sources and a double Fourier integral:

$$G = G^R + G^F \tag{23a}$$

In deep water, the Rankine component G^R may be taken as

$$G^R = \frac{-1}{4\pi}\left(\frac{1}{r} - \frac{1}{r'}\right) \tag{23b}$$

where the Rankine sources r and r' are defined by (10a) and (16a). The corresponding Fourier component G^F for deep water is given by the double Fourier integral

$$G^F = \frac{1}{4\pi^2}\lim_{\varepsilon\to+0}\int_{-\infty}^{\infty} d\beta \int_{-\infty}^{\infty} d\alpha\;\frac{e^{\,i\,[\,\alpha\,(x-\xi)+\beta\,(y-\eta)\,]+k\,(z+\zeta)}}{(f - F\alpha)^2 - k + i\,\varepsilon\,(f - F\alpha)} \tag{23c}$$

e.g. see *Noblesse (2001a)*. Here, $k=\sqrt{\alpha^2+\beta^2}$ is the wavenumber associated with the Fourier variables α and β. By substituting the decomposition (23a) of the Green function into the potential representations (20a) and (20b), we can express ϕ_* as

$$\phi_* = \phi_*^R + \phi_*^F \tag{24}$$

The contributions of the Rankine component G^R to the surface integrals over the mean free surface Σ_0 and the line integrals along the boundary curve Γ in the potential representations (20a) and (20b) are null, and the Rankine component ϕ_*^R in the Rankine-Fourier decomposition (24) of the potential ϕ_* is given by

$$\phi_*^R = \frac{-1}{4\pi}\int_{\Sigma_B} d\mathcal{A}\,[\,u^n\,(\frac{1}{r}-\frac{1}{r'}) + \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}] \tag{25a}$$

Here, a_1 is associated with the classical potential representation (20a) and is defined as

$$a_1 = -\phi\;\vec{n}\cdot\nabla\,(\frac{1}{r}-\frac{1}{r'}) \tag{25b}$$

The function a_2 corresponds to the weakly-singular representation (20b) and is given by

$$a_2 = (\Omega^y X - \Omega^x\,Y)(\frac{\mathrm{sign}(Z)}{r\,(r+|\,Z\,|)} + \frac{1}{r'\,(\,r'-Z'\,)}) \tag{25c}$$

in accordance with (17). Here, $X, Y, Z\,, Z', r$ and r' are given by (10a) and (16a). Expressions (25), (10a) and (16a) show that the Rankine potential ϕ_*^R is null at the mean free surface $\zeta=0$. Thus, (24) becomes

$$\phi_* = \phi_*^F \quad \text{at} \quad \zeta = 0 \tag{26}$$

By substituting expression (23c) for the Fourier component of the Green function into (20a) and (20b), we obtain the Fourier-Kochin representation of the free-surface potential ϕ_*^F

$$\phi_*^F = \frac{1}{4\pi^2}\lim_{\varepsilon\to+0}\int_{-\infty}^{\infty} d\beta \int_{-\infty}^{\infty} d\alpha\; \frac{e^{-i\,(\,\alpha\,\xi\,+\,\beta\,\eta\,)\,+\,k\,\zeta}\,S(\alpha\,,\beta)}{(f-F\alpha)^2 - k + \,i\,\varepsilon\,(f-F\alpha)} \tag{27}$$

The amplitude (or spectrum) function $S(\alpha\,,\beta)$ in the Fourier representation (27) is given by distributions of elementary waves along the boundary curve Γ and over the boundary surface Σ_B and the free surface Σ_0. Specifically, the spectrum function S is given by

$$\begin{aligned} S = &\int_{\Sigma_B} d\mathcal{A}\,[\,u^n - (\,i\,\alpha\,n^x + i\,\beta\,n^y + k\,n^z\,)\,\phi\,]\,e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)\,+\,k\,z} \\ &-\int_{\Sigma_0} dx\,dy\;\Phi\;e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)} + \int_{\Gamma} d\mathcal{L}\,[\,F^2(u - i\,\alpha\,\phi) + i\,\widehat{\tau}\,\phi\,]\,t^y\,e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)} \end{aligned} \tag{28a}$$

$$\begin{aligned} S = &\int_{\Sigma_B} d\mathcal{A}\,(u^n + i\,\frac{\alpha\,\Omega^y - \beta\,\Omega^x}{k})\,e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)\,+\,k\,z} - \int_{\Sigma_0} dx\,dy\;\Phi\;e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)} \\ &+\int_{\Gamma} d\mathcal{L}\,[\,F^2(t^y u + \frac{\alpha\beta}{k^2}\,\phi_t) - \widehat{\tau}\,\frac{\beta}{k^2}\,\phi_t + i\,f^2\,\frac{\alpha\,t^y - \beta\,t^x}{k^2}\,\phi\,]\,e^{\,i\,(\,\alpha\,x\,+\,\beta\,y\,)} \end{aligned} \tag{28b}$$

for the classical representation (20a) and the weakly-singular representation (20b), respectively. Expression (26) and the Fourier-Kochin representation of the potential ϕ_*^F given by (27) and (28) show that, at the free surface $\zeta=0$, differences between the classical potential representation (20a) and the weakly-singular representation (20b) stem from the corresponding spectrum functions (28a) and (28b). Expression (28b), obtained here from the weakly-singular potential representation (20b), was previously obtained in *Noblesse and Yang (1995)* via a transformation of (28a).

The alternative spectrum functions (28a) and (28b) are compared in *Noblesse et al. (2002)*, where the steady flow generated by a point source and a point sink at an ellipsoidal boundary surface Σ_B is considered. This comparison shows that the numerical cancellations that occur between the surface integral over the boundary surface Σ_B and the line integral along the boundary curve Γ in expression (28a) for the spectrum function associated with the classical potential representation (20a) are largely eliminated in expression (28b) for the spectrum function associated with the weakly-singular potential representation (20b).

10. Velocity at boundary surface and at boundary curve

The potential representations (12b), (13b), (14b), (20b) involve the tangential components Ω^x, Ω^y and u, v of the velocity $\vec{u}$. These tangential velocity components are now expressed in terms of the potential ϕ at the boundary surface Σ. Let $\vec{s} = (s^x, s^y, s^z)$ and $\vec{t} = (t^x, t^y, t^z)$ stand for two unit vectors tangent to Σ. The unit vector $\vec{n} = (n^x, n^y, n^z)$ normal to Σ is related to the tangent vectors $\vec{s}$ and $\vec{t}$ by

$$\vec{n} = (\vec{s} \times \vec{t}\,)/\sqrt{1-\mu^2} \qquad \text{with} \qquad \mu = \vec{s} \cdot \vec{t} \tag{29}$$

The velocity $\vec{u}$ at the surface Σ can be expressed as

$$\vec{u} = u^n\,\vec{n} + u^s\,\vec{s} + u^t\,\vec{t} \tag{30a}$$

where $(u^n, u^s, u^t\,)$ are the components of $\vec{u}$ along the unit vectors $\vec{n}\,,\vec{s}\,,\vec{t}$ attached to Σ. The components u^s and u^t in (30a) can be expressed in terms of the derivatives ϕ_s and ϕ_t of the potential ϕ along the unit tangent vectors $\vec{s}$ and $\vec{t}$. Specifically, (30a) yields

$$\phi_s = \vec{u} \cdot \vec{s} = u^s + \mu\, u^t \qquad\qquad \phi_t = \vec{u} \cdot \vec{t} = u^t + \mu\, u^s$$

We then have

$$u^s = \frac{\phi_s - \mu\,\phi_t}{1-\mu^2} \qquad\qquad u^t = \frac{\phi_t - \mu\,\phi_s}{1-\mu^2} \tag{30b}$$

and (30a) becomes

$$\vec{u} = u^n\,\vec{n} + \phi_s\,\frac{\vec{s} - \mu\,\vec{t}}{1-\mu^2} + \phi_t\,\frac{\vec{t} - \mu\,\vec{s}}{1-\mu^2} \tag{30c}$$

Expressions (30c) and (29) show that the vector $\vec{\Omega} = \vec{u} \times \vec{n}$ is given by

$$\vec{\Omega} = \frac{\phi_t\,\vec{s} - \phi_s\,\vec{t}}{\sqrt{1-\mu^2}} = \Omega^s\,\vec{s} + \Omega^t\,\vec{t} \tag{31a}$$

It follows that

$$\Omega^x = \frac{s^x \phi_t - t^x \phi_s}{\sqrt{1-\mu^2}} \qquad\qquad \Omega^y = \frac{s^y \phi_t - t^y \phi_s}{\sqrt{1-\mu^2}} \tag{31b}$$

Expressions (29)–(31) hold for the body surface Σ_B and the free surface Σ_0.

The unit vectors $\vec{s}$ and $\vec{t}$ tangent to the body surface Σ_B must be oriented so that the unit vector $\vec{n}$ defined by (29) points into the flow domain. Furthermore, at the boundary curve Γ, the vector $\vec{t}$ is tangent to Γ and oriented clockwise (looking down). Thus, the unit vector $\vec{s}$ tangent to Σ_B points upward along the curve Γ. In the special case of a body surface Σ_B that intersects the free-surface plane orthogonally, we have $\vec{s} = (0\,,0\,,1)$ at the intersection curve Γ of Σ_B with the free surface, and expressions (31b) yield $(\,\Omega^x, \Omega^y\,) = -(\,t^x, t^y\,)\,w$.

At the free-surface plane Σ_0, the horizontal velocity components u and v can be expressed in terms of the derivatives ϕ_s and ϕ_t of the potential ϕ along two unit vectors $\vec{s} = (s^x, s^y, 0)$ and $\vec{t} = (t^x, t^y, 0)$ in the plane $z=0$. Specifically, (2b) and (31b) yield $\Omega^y = u$, $\Omega^x = -v$, and

$$u = \frac{s^y \phi_t - t^y \phi_s}{\sqrt{1-\mu^2}} \qquad\qquad -v = \frac{s^x \phi_t - t^x \phi_s}{\sqrt{1-\mu^2}} \tag{32a}$$

The unit vectors $\vec{s}$ and $\vec{t}$ in the free-surface plane Σ_0 must be oriented so that (29) defines a unit vector $\vec{n}$ normal to Σ_0 that points into the flow domain; i.e. we must have $\vec{n} = (0, 0, -1)$. In addition, at the boundary curve Γ, the vector $\vec{t}$ is tangent to Γ and oriented clockwise as already noted. Thus, along the curve Γ, the unit vector $\vec{s}$ tangent to Σ_B points into the flow domain (like the vector $\vec{n}$). At the curve Γ, the vector $\vec{s}$ in (32a) may be taken as the unit vector $\vec{\nu} = (-t^y, t^x, 0)$ normal to the curve Γ in the plane Σ_0. Expressions (32a) then become

$$u = t^x \phi_t - t^y \phi_\nu \qquad\qquad v = t^x \phi_\nu + t^y \phi_t \tag{32b}$$

The potential representations (20a), (20b), and (22) involve u (the $x-$component of the velocity $\vec{u}$) at the boundary curve Γ. The expression

$$u = t^x \phi_t - t^y\, \phi_\nu \tag{33a}$$

given by (32b) defines u at the curve Γ in terms of derivatives of the potential ϕ along two unit vectors $\vec{t} = (t^x, t^y, 0)$ and $\vec{\nu} = (-t^y, t^x, 0)$ that lie in the free surface Σ_0. At the body surface Σ_B, expressions (30c), (30b), and (29) yield

$$u = t^x \phi_t + n^x u^n + t^y n^z \sqrt{1-\mu^2}\, u^s \tag{33b}$$

This expression defines u at the curve Γ in terms of derivatives of ϕ along two unit vectors $\vec{t} = (t^x, t^y, 0)$ and $\vec{s} = (s^x, s^y, s^z)$ that are tangent to the body surface Σ_B. Expression (33b) is given in *Brard (1972)* and has been used in numerous numerical and theoretical studies, including the Fourier-Kochin approach developed in *Noblesse and Yang (1995)* and the slender-ship approximation to steady ship waves given in *Noblesse (1983)*. Expressions (33a) and (33b) yield identical values of u at Γ if

$$-t^y \phi_\nu = n^x u^n + t^y n^z \sqrt{1-\mu^2}\, u^s$$

In the special case of a body surface Σ_B that intersects the free surface orthogonally, we have $n^z = 0$ and $n^x = -t^y$, and the foregoing relation becomes $\phi_\nu = u^n = \vec{u} \cdot \vec{n} = \phi_n$. Thus, the "free-surface expression" (33a) and the "body-surface expression" (33b) yield identical values of the velocity component u at the boundary curve Γ if the boundary condition $\phi_n = u^n$ at the body surface Σ_B is satisfied at the free-surface plane $z=0$. It is not a priori obvious that this continuity condition holds because the boundary conditions at the body surface Σ_B and at the free surface Σ_0 may not allow a continuous velocity field at the intersection curve Γ.

11. Conclusion

The boundary-integral representations (9), (11), (12b), (13b), (14b), (20b), (21b), (22b) define the potential ϕ_* at a field point $\vec{\xi}$ in terms of a Green function G and related functions that are no more singular than G. These potential representations are weakly singular in comparison to the classical boundary-integral representations (3), (12a), (13a), (14a), (20a), (21a), (22a), which define the potential in terms of G and ∇G. Another interesting difference between the classical potential representations and the corresponding weakly-singular representations is that the latter representations define a potential ϕ_* that is continuous at the boundary surface Σ (the potential defined by the classical representations, which involve surface distributions of dipoles, is well-known to be discontinuous at the boundary surface).

The illustrative numerical application to the steady flow due to a point source and a point sink presented in *Noblesse et al. (2002)* shows that the numerical cancellations (and related loss of accuracy) that occur between the waterline integral and the hull integral in the classical representation (22a) of steady ship waves are largely eliminated in the weakly-singular representation (22b). This property indicates that the weakly-singular potential representations given in this study may be useful for numerical purposes.

Expressions (31b) and (32) — which define the tangential velocity components Ω^x, Ω^y and $u\,, v$ in terms of the derivatives ϕ_s and ϕ_t of the potential ϕ along unit vectors $\vec{s}$ and $\vec{t}$ tangent to the boundary surface $\Sigma_B \cup \Sigma_0$ — show that the weakly-singular potential representations (12b), (20b), (21b) and (22b) provide integro-differential equations that can be used to determine the potential at $\Sigma_B \cup \Sigma_0$; similarly, the potential representations (11), (13b) and (14b) provide integro-differential equations that define the potential at Σ_B . In practice, the weakly-singular potential representations can be enforced at a set of collocations points $\vec{\xi}$ taken as vertices of a set of flat triangular panels that closely approximate the boundary surface. In this low-order panel method, the potential ϕ is assumed to vary linearly within a panel, and the tangential velocity components Ω^x, Ω^y (on body-surface panels) and $u\,, v$ (on free-surface panels) are constant within a panel, and are determined from the values of the potential at the vertices of the triangular panel using (31b) and (32a). On body-surface panels, the normal velocity u^n (known from the boundary condition at the body surface Σ_B) is likewise constant. Thus, the potential ϕ varies linearly and both the normal and tangential components of the velocity $\vec{u}$ are constant within a panel (a consistent level of approximation).

An important difference between the weakly-singular representation (7) of the velocity, extended to free-surface flows in *Noblesse (2001b)*, and the representation (9) of the potential is that the velocity representation (7) does not ensure potential flow. In addition, the approach expounded in *Noblesse (2001b)* for free-surface flows, based on the classical potential representation (9) and the weakly-singular velocity representation (7), yields a system of four coupled representations for the potential ϕ and the three components $u\,, v\,, w$ of the velocity $\vec{u}$. These representations of the potential and the velocity involve G and ∇G . The weakly-singular potential representation (9), on the other hand, provides a single (integro-differential) representation that defines ϕ and $\vec{u} = \nabla\phi$ and that only involves G and related functions no more singular than G , as explained above.

In the particular case $f = 0$ (i.e. for steady flows), the integral over Σ_B in the weakly-singular potential representation (22b) involves u^n, Ω^x and Ω^y, which are defined in terms of the velocity $\vec{u}$ by (2), and the integral over Γ involves u and $\phi_t = \vec{u} \cdot \vec{t}$. Thus, if $f = 0$, the integrals over Σ_B and Γ in the representation (22b) only involve the velocity $\vec{u}$; i.e. these integrals do not involve the potential ϕ , unlike the classical representation (22a).

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