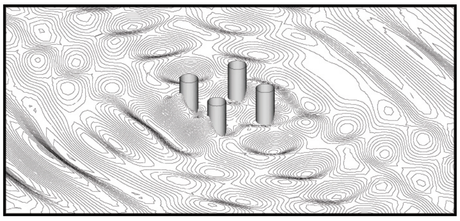

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NONLINEAR MODELLING OF LARGE AMPLITUDE SHIP ROLLING IN LONGITUDINAL WAVES

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Summary

The nonlinear modelling of parametric rolling of a ship advancing in longitudinal waves is studied on an experimental basis. Different descriptions of the righting arm in waves are compared with experimental results using a perturbative analytical solution obtained by means of the averaging method. It appears that an isolated roll motion equation provides a very reliable simulation of experimental results including both threshold and roll amplitude above threshold, but the interaction between righting arm nonlinearity and time dependence induced by wave and vertical motions requires further studies.

I - INTRODUCTION

A number of researchers investigated the parametrically excited rolling in following seas since the coupling between roll motion and vertical motions can lead to dramatic roll amplitudes and possibly capsizing. The results were successively generalised to the case of quartering waves and to phenomena different from parametric excitation as pure loss of stability, and connections between stability and manoeuvrability as surf-riding and broaching. As a result, the diagram containing information on dangerous combinations of heading and ship velocity is now part of recommendations of international intact stability rules.

The parametric rolling in head waves is a less studied phenomenon since the merchant ships more affected by this phenomenon encounter the waves with the required tuning ratio in following sea conditions, where, in addition, the relative ship speed is low, manoeuvrability is poor and roll damping is also low. On the contrary, in the head sea condition, relative ship velocity is high and consequently the roll damping increases for the effect of advance speed.

Preliminary calculations done on the scale model of a destroyer, already extensively studied in beam waves to identify the extreme non-linear dynamics, revealed that the tuning condition more appropriate for parametric rolling excitation in longitudinal waves with wavelength equal to ship length at waterline, i.e. $\omega_e/\omega_f=2$ could be met preferably in head waves in operative loading conditions.

The model was thus 'attached' to the towing tank carriage by means of a tethering system based on couples of elastic mooring lines symmetric with respect to the centreline to keep the model course and sufficiently loose to avoid any interference with the roll motion and with the vertical motions. The model was then run at speed in a wide interval of velocities in head regular waves with wavelength equal to model length at waterline. Several wave heights were tested to investigate the detailed structure of the threshold for parametric rolling, its dependence on the tuning ratio and the roll motion amplitude above threshold. This last is an important item since usually we read that above threshold we can have roll instability, but rarely we read in the reports that this rolling motion, when excited, usually is bounded and rarely unbounded, this last fact appearing as a kind of second threshold. The roll decays at speed were also recorded together with the vertical acceleration and the pitch angle to reconstruct the ship relevant parameters for the simulation of the phenomenon. In particular, frequent cases of green water on deck were observed at the highest sea state (wave steepness=1/30) were observed. The onset of very large roll amplitude leading to capsize was restricted to the highest sea state and to the lower ship speeds. The correlation of these results to the dependence of roll damping on ship speed and between the thresholds and the relative variations of righting arm in waves computed by matching the results of strip theory and a hydrostatic calculations was satisfactory.

The problem of the prediction of roll amplitude above threshold was solved by means of a nonlinear modelling taking into account the nonlinear features of the righting arm and the nonlinear features of the difference between maximum and minimum value of the righting arm in waves, which was seen to disappear at large angles. The comparison between experimental and approximate perturbative solution of the equation of motion was satisfactory.

II - PARAMETRIC ROLLING

The description of ship rolling in a purely longitudinal sea can be obtained by considering the following mathematical model:

$$I'\ddot{\phi}+D(\dot{\phi},\phi)+R(\phi,t)=0 \qquad (1)$$

where I' is the virtual moment of inertia, D the damping, $R=\Delta\overline{GZ}(\phi)$ the restoring and there is no explicit forcing term due to the wave action. Actually, all terms become time dependent, but explicit time dependence is usually retained only in the righting moment R, being the other of minor entity. Both D and R are non-linear and display this feature at large amplitudes of the motion. If we consider the onset of parametric rolling, that is if we restrict for the moment to the analysis of the stability of the solution $\phi(t)\equiv 0$, the Eq. 1 can be linearised, partly simplifying the problem:

$$I'\ddot{\phi}+M\dot{\phi}+\Delta\overline{GM}(t)\phi=0 \qquad (2)$$

Considering a sinusoidal time variation of the transversal metacentric height (otherwise we can consider the first term of its Fourier series) with amplitude $\delta\overline{GM}$ around the average value $\overline{GM}*$ and, dividing as usual by I', one has:

$$\ddot{\phi}+2\mu\dot{\phi}+\omega_0^2\left[1+\frac{\delta\overline{GM}}{\overline{GM}*}\cos(\omega_e t+\varepsilon)\right]\phi=0 \qquad (3)$$

which is an equation of the Mathieu type [1]. The phase ε can be neglected without prejudice for the subsequent analysis and with a change of the time scale $\omega_e t = 2t'$ (retaining the same name) Eq. 3 can be transformed into the more familiar form:

$$\ddot{\phi} + 2\mu^* \dot{\phi} + 4\frac{\omega_0^2}{\omega_e^2}\left[1 + \frac{\delta\overline{GM}}{\overline{GM}^*}\cos(2t)\right]\phi = 0 \tag{4}$$

with $\mu^* = 2\mu / \omega_e$.

Cancelling the damping by means of a linear transformation, which introduces an $e^{-\mu^* t}$ amplitude change in the roll amplitude, with $\mu^* = \dfrac{2\mu}{\omega_e}$, and considering that the natural frequency of the slightly damped rolling $\omega_{0damp} \cong \omega_0$ we obtain

$$\ddot{\phi} + 4\frac{\omega_0^2}{\omega_e^2}\left[1 + \frac{\delta\overline{GM}}{\overline{GM}^*}\cos(2t)\right]\phi = 0 \tag{5}$$

This equation is a differential equation with periodic coefficients. Floquet theory [1] indicates that its solutions can be put in the form $e^{\pm\sigma t}\psi(t)$ where $\psi(t)$ is a periodic function and σ is the “characteristic exponent". The solutions of undamped Mathieu equation are thus diverging if $\sigma \neq 0$ and stable if σ=0. If linear damping is present, the situation is qualitatively and quantitatively modified, i.e. the solutions of damped Mathieu equation and the ship behaviour will be:

Condition	Mathieu eqn solution	Ship vertical position equilibrium	Effect of a perturbation to Ship vertical position
both $-\mu^* \pm \sigma > 0$	Diverging	Stable	Growing in time
$\mu^* = \sigma = 0$	Stable	Unstable	Stable in time
both $-\mu^* \pm \sigma < 0$	Decaying	Unstable	Decaying in time

These conclusions are valid to discuss the stability of equilibrium and hence initial stability. They are valid to discuss roll amplitude boundedness and hence dynamic ship stability (or stability in the large) as far as the ship rolling can be described by a linear mathematical model like Eq. 1 above. From a practical point of view, we know that this is no longer true at large angles, where Eq. 1 has to be substituted by a much more complicated non-linear model, which will be solved by means of a perturbation method.
The possibility of onset of the dangerous phenomenon of parametric rolling is thus tied to the simultaneous verification of the following conditions:

- the ratio of the encounter wave frequency ω_e to the natural roll frequency ω_0 is close to the condition:
  $\dfrac{\omega_e}{\omega_0} \approx \dfrac{2}{n}$ with n integer; (6)
- the periodic variation of metacentric height due the combined effect of ship motions and wave is sufficiently large;

- the roll damping is sufficiently small.

As far as the first instability zone is concerned (n=1), a threshold value:

$$\frac{\delta\overline{GM}}{\overline{GM}*} = \sqrt{\left(2 - \frac{\omega_e^2}{\omega_0^2}\right)^2 + 2\frac{\omega_e^4}{2\omega_0^4}\left(\frac{4\omega_0^2}{\omega_e^2} + 1\right)(\mu *)^2} \qquad (7)$$

for the onset of parametric rolling, is found, which reduces to

$$2 - \frac{\omega_e^2}{2\omega_0^2} < \frac{\delta\overline{GM}}{\overline{GM}*} < \frac{\omega_e^2}{2\,\omega_0^2} - 2 \qquad (8)$$

when $\mu * = 0$, while it gives a minimum threshold value

$$\frac{\delta\overline{GM}}{\overline{GM}*} = 4\mu * = \frac{8\mu}{\omega_e} = \frac{4\mu}{\omega_0} \qquad (9)$$

in proximity of the "exact" synchronism condition $\omega_e = 2\omega_0$.

When a nonlinear damping term is considered, e.g. viscous damping, a threshold similar to Eq. 7-9 can still be obtained [2].

III - ROLL MOTION AMPLITUDE ABOVE THRESHOLD

The threshold phenomenon is known since the early fifties. On the other hand, the mathematical modelling of the roll motion above threshold is still subject of discussions. A nonlinear approach, based on a series of experiments conducted on the scale model of a destroyer is here presented. The head sea condition was tested due to the fact that experiments have been conducted in the towing tank of the Department of Naval Architecture of the University of Trieste. The limited length of the towing tank (50 m) makes possible experiments in following waves only at very low speed.

In Table. 1 the main dimensions and mechanical data of the ship model are presented. The righting arm in full scale in calm water is represented in Fig. 1.

Lbp	(2.532±0.001) m
Loa	(2.640±0.001) m
B	(0.273±0.001) m
T	(0.080±0.001) m
Δ	(260±1) N
$\overline{KG}$	(0.134±0.002) m
$\overline{GM}$	(0.015±0.002) m
ω_0 (at zero speed)	(3.42±0.03) rad/s
μ_1	0.0752
μ_2	0.0896

Table 1. Main data of the scale model of the destroyer.

Two series of experiments have been conducted:

- roll decay in calm water with forward speed to obtain an estimate of roll frequency and of roll damping dependence on velocity. The results have been analysed in terms of equivalent linear damping, which resulted to be well approximated by a cubic function of the forward speed: $\mu = \mu_1 + \mu_2 v^3$;
- analysis of the stability of vertical position versus excitation of parametric rolling with forward speed in head waves with wavelength equal to ship length at waterline. Three different wave steepnesses have been tested: $s_w = \frac{h_w}{\lambda_w} = \frac{1}{100}, \frac{1}{50}, \frac{1}{30}$.

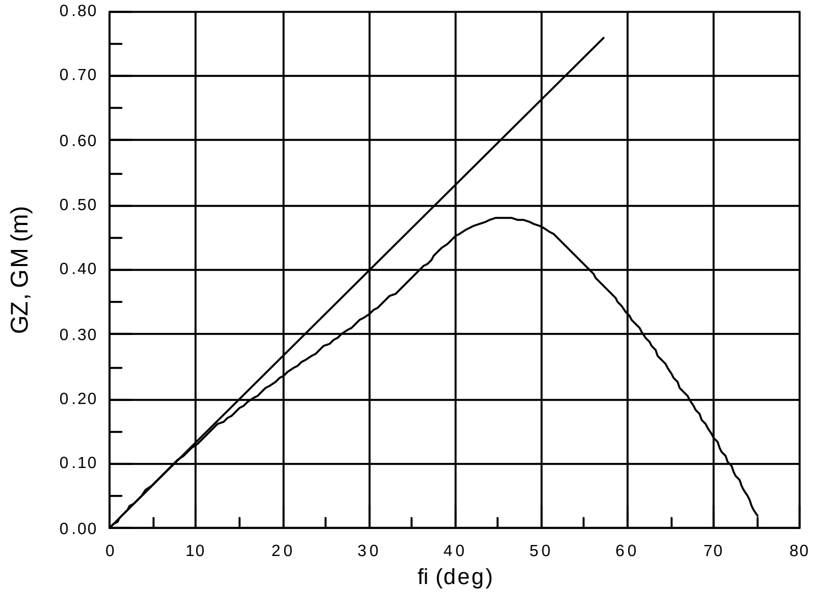

Fig. 1. Righting arm in calm water.

The details of experimental setup and the experimental results are reported in [3-5]. In Fig. 2 and Fig. 3, the righting arm with crest/through amidships are compared with the calm water result for the two extreme wave steepnesses. The righting arms have been calculated in the free trim hydrostatic equilibrium hypothesis.
Looking for a mathematical modelling of the parametric roll, we first consider a model based on one ordinary differential equation describing isolated rolling motion with the inclusion of one or more time dependent terms describing the interaction with the waves.
The possibility of using concentrate parameter models to simulate large amplitude motions has been the subject of many discussions in the past. The conclusion is that the separability of the different contributions (added mass, damping, restoring and forcing) is possible only in the presence of small amplitude rolling motion [6]. The same is true for the coupling between rolling motion and the other lateral motions. From a practical point of view, an extensive series of measurements conducted on several scale models in beam waves of small and so small steepness, with roll amplitudes attaining 40 deg in several cases, indicated that the possibility of a reliable description based on isolated roll motion differential equation goes far beyond the expentance [7-9]. This is something similar to the question of the validity of the results of perturbation methods applied to nonlinear roll motion which, although based on the hypotheses of small perturbation parameter (connected usually with amplitudes <<1), compare reasonably well with exact numerical results extending to very large amplitudes.
In the following, therefore, we will try again the same route paved with the following assumptions:
- separability of calm water and wave actions;

- single degree of freedom;
- applicability of perturbation method to obtain reliable approximate analytical solutions;
- effect of longitudinal wave on added mass and damping negligible with respect to the effect on righting arm.

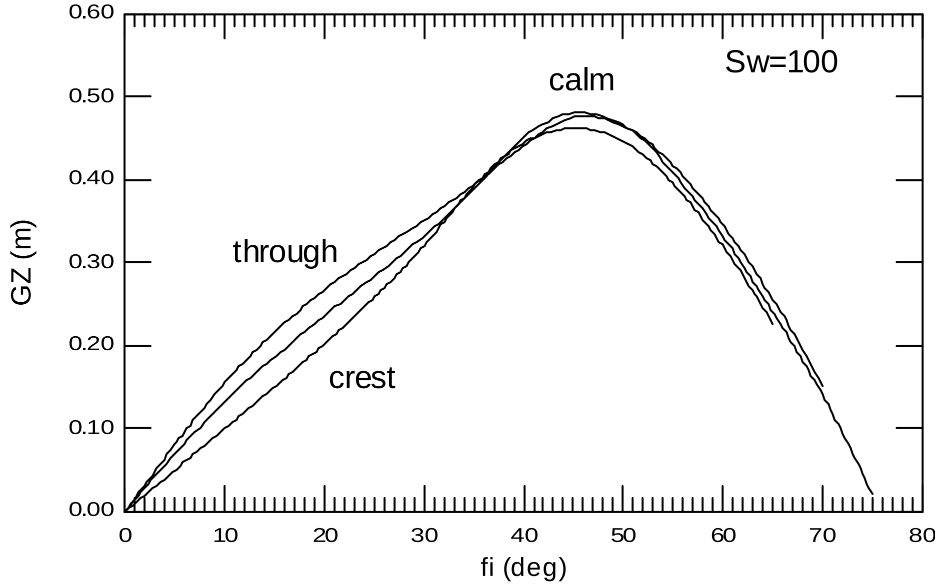

Fig. 2. Righting arm in calm water and in the presence of a 1/100 steepness longitudinal wave.

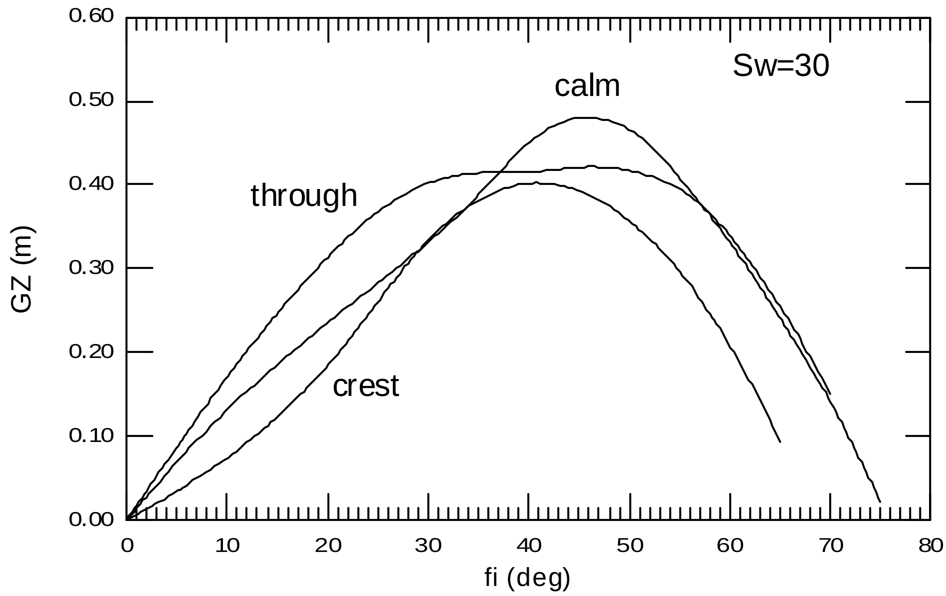

Fig. 3. Righting arm in calm water and in the presence of a 1/100 steepness longitudinal wave.

The only contribution of the longitudinal wave will therefore be on righting arm: different models for the description of this action will be proposed in the following. We have the intrinsic nonlinearity in the angle of the righting arm (that is present also in calm water) and the time variation of the righting arm depending on the longitudinal wave passing along the ship and its direct effects (vertical motions). These two effects can be considered in "coupled" and "uncoupled" models.

The analysis of the curves relative to crest and through of the wave amidships reveals that the righting arm oscillation is given by the linear approximation:

$$\overline{GM}^{*} p_1 \phi \cos \omega_e t \tag{10}$$

with $p_1 = \dfrac{\delta\overline{GM}}{\overline{GM}*}$ at small inclinations, then it grows to a maximum value and finally vanishes at an angle $\phi_{max} \approx 35 \div 40°$. On this basis, a parabolic variation of the amplitude of this oscillation was assumed.

III.1 - Uncoupled models

The following mathematical model was selected for this preliminary nonlinear approach:

$$\ddot{\phi} + 2\mu\dot{\phi} + \delta\dot{\phi}^3 + \left[1 + \left(p_1 + p_2\phi^2\right)\cos \omega_e t\right]\omega_0^2 \phi + \alpha_3 \phi^3 + \alpha_5 \phi^5 + \ldots = 0 \tag{11}$$

with $p_2 < 0$.
In Eq. 11, the representation of the righting moment through a polynomial was used:

$$m_R = \frac{R}{I'} = \omega_0^2 \phi + \alpha_3 \phi^3 + \alpha_5 \phi^5 \ldots \tag{12}$$

The values of the coefficients $\alpha_3, \alpha_5, \alpha_7, \ldots$ can be obtained by means of a least square fit to the hydrostatic calculation results. In this study only the cubic nonlinear term will be retained. Posing again: $\omega_e t = 2t'$ and retaining the same name for time:

$$\ddot{\phi} + 2\mu^* \dot{\phi} + \delta^* \dot{\phi}^3 + \left[1 + \left(p_1 + p_2\phi^2\right)\cos 2t\right] \omega_0^{*2} \phi + \alpha_3^* \phi^3 = 0 \tag{13}$$

with: μ^* as above and $\delta^* = \delta \dfrac{\omega_e}{2}$ $\alpha_3^* = \alpha_3 \dfrac{4}{\omega_e^2}$ $\omega_0^{*2} = \dfrac{4\omega_0^2}{\omega_e^2}$.

The same mathematical model, with parametric excitation represented by p_1 only has been used by other authors (see for instance [10]).
In the first instability zone n=1 and $\omega_e \approx 2\omega_0$ the solution has the form:

$$\phi(t) \approx A \sin t + B \cos t \tag{14}$$

with A and B “slowly varying” amplitudes.
Deriving, substituting in Eq. (8) and using the auxiliary condition:

$$\dot{A} \cos t + \dot{B} \sin t = 0 \tag{15}$$

a system of algebraic equations is obtained for A and B. Averaging over one period, the following evolutionary system is obtained for the averaged time derivatives $\langle \dot{A} \rangle$ and $\langle \dot{B} \rangle$:

$$2\langle\dot{A}\rangle=-\left(\mu^{*}+\frac{3}{4}\delta^{*}\left(A^{2}+B^{2}\right)\right)A+B\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^{*}\left(A^{2}+B^{2}\right)\right)-2p_1\frac{\omega_0^2}{\omega_e^2}B+2p_2\frac{\omega_0^2}{\omega_e^2}B^{3}$$

(16)

$$2\langle\dot{B}\rangle=-\left(\mu^{*}+\frac{3}{4}\delta^{*}\left(A^{2}+B^{2}\right)\right)B-A\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^{*}\left(A^{2}+B^{2}\right)\right)-2p_1\frac{\omega_0^2}{\omega_e^2}A+2p_2\frac{\omega_0^2}{\omega_e^2}A^{3}$$

The stationary solution $C=\sqrt{A^2+B^2}$ can then be obtained by solving for A and B the above system with the position $\langle\dot{A}\rangle=\langle\dot{B}\rangle=0$. Since this is quite complicated for insertion in a parameter identification technique for a nonlinear system in the presence of bifurcations [11], in this paper a simplified approach was effectively used, based on the use of an "average" p-value:

$$p_{ave}=p_1+\frac{p_2}{3}\phi^2 \tag{17}$$

at the generic iteration of the zero-searching procedure used to solve the algebraic equation giving C. Adopting a "constant" p-value, indeed, the above algebraic system reduces quite easily to a single algebraic second degree equation for C, which can be easily solved.

III.2 - Coupled system

Now the effects of the wave passing along the ship and the intrinsic nonlinearity of the righting arm are coupled:

$$\ddot{\phi}+2\mu^{*}\dot{\phi}+\delta^{*}\dot{\phi}^{3}+\left[1+\left(p_1+p_2\phi^2\right)\cos 2t\right]\left(\omega_0^{*2}\,\phi+\alpha_3^{*}\phi^{3}\right)=0 \tag{18}$$

By applying the same procedure as before, the evolutionary equations of the solution are given by:

$$2\langle\dot{A}\rangle=-\left(\mu^{*}+\frac{3}{4}\delta^{*}\left(A^{2}+B^{2}\right)\right)A+B\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^{*}\left(A^{2}+B^{2}\right)\right)-\frac{p_1}{2}\left(4\frac{\omega_0^2}{\omega_e^2}B+\alpha_3^{*}B^{3}\right)$$
$$+\frac{p_2}{2}\left(4\frac{\omega_0^2}{\omega_e^2}B^{3}+\alpha_3^{*}\left[-\frac{15}{16}B^{5}+\frac{5}{16}A^{4}B-\frac{5}{8}A^{2}B^{3}\right]\right)$$

(19)

$$2\langle\dot{B}\rangle=-\left(\mu^{*}+\frac{3}{4}\delta^{*}\left(A^{2}+B^{2}\right)\right)B-A\left(\left(4\frac{\omega_0^2}{\omega_e^2}-1\right)+\frac{3}{4}\alpha_3^{*}\left(A^{2}+B^{2}\right)\right)-\frac{p_1}{2}\left(4\frac{\omega_0^2}{\omega_e^2}A+\alpha_3^{*}A^{3}\right)$$
$$+\frac{p_2}{2}\left(4\frac{\omega_0^2}{\omega_e^2}A^{3}+\alpha_3^{*}\left[\frac{15}{16}A^{5}-\frac{5}{16}AB^{4}+\frac{5}{8}A^{3}B^{2}\right]\right)$$

Which gives a fully nonlinear system of algebraic equations in A and B for the stationary solution.

IV - COMPARISON WITH EXPERIMENTAL RESULTS AND CONCLUSIONS

The experimental results are given in Fig. 4 to 6. The simulation was obtained by using Eq. 16 with the simplifying hypothesis (17). The parameter values used were:
- the experimentally measured values for damping;
- the value of p_2 obtained from free trim hydrostatic analysis;

whereas the values of p_1 and α_3 have been estimated by means of a nonlinear regression to the experimental values (Parameter Identification Technique).

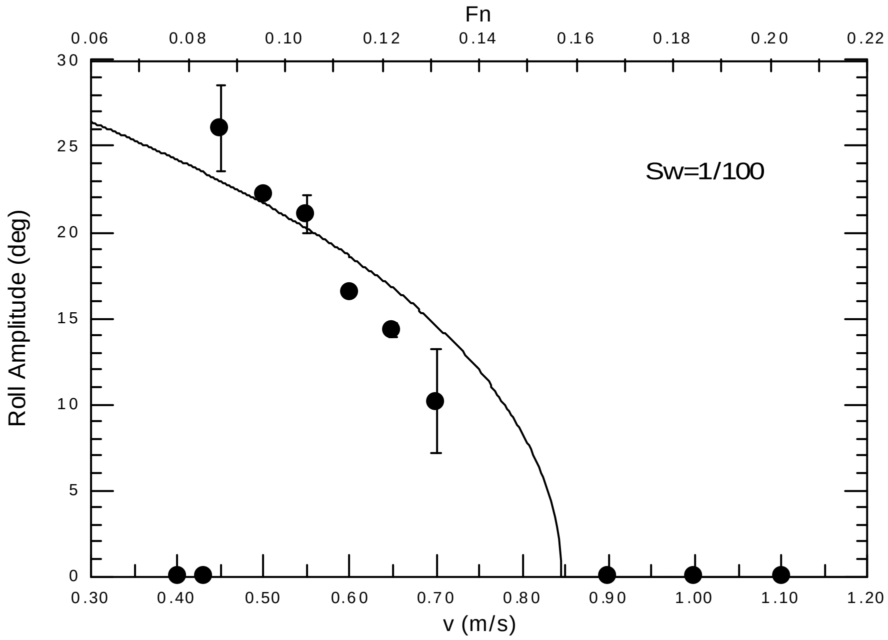

Fig. 4. Experimental results versus simulation based on Eq. 16-17 for the case s_w=1/100.

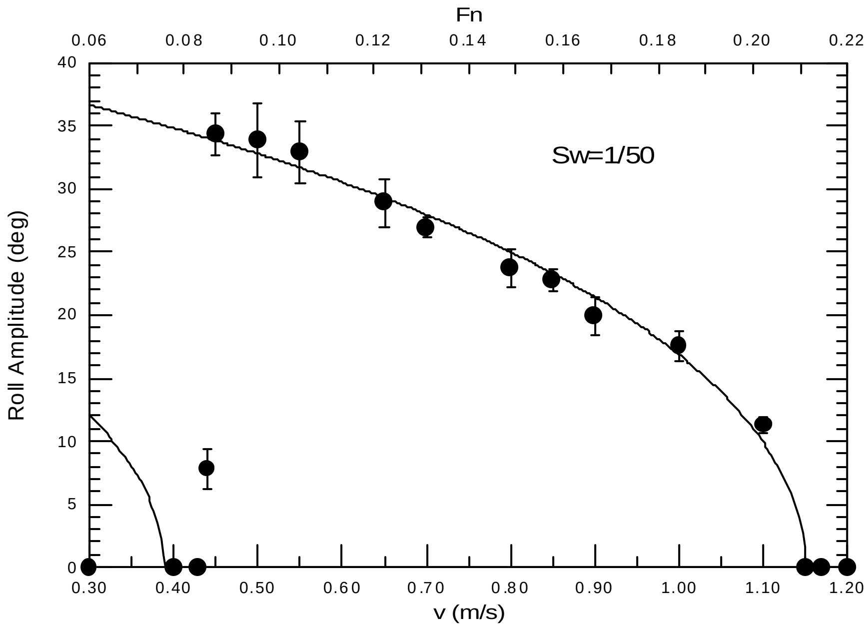

Fig. 5. Experimental results versus simulation based on Eq. 16-17 for the case s_w=1/50.

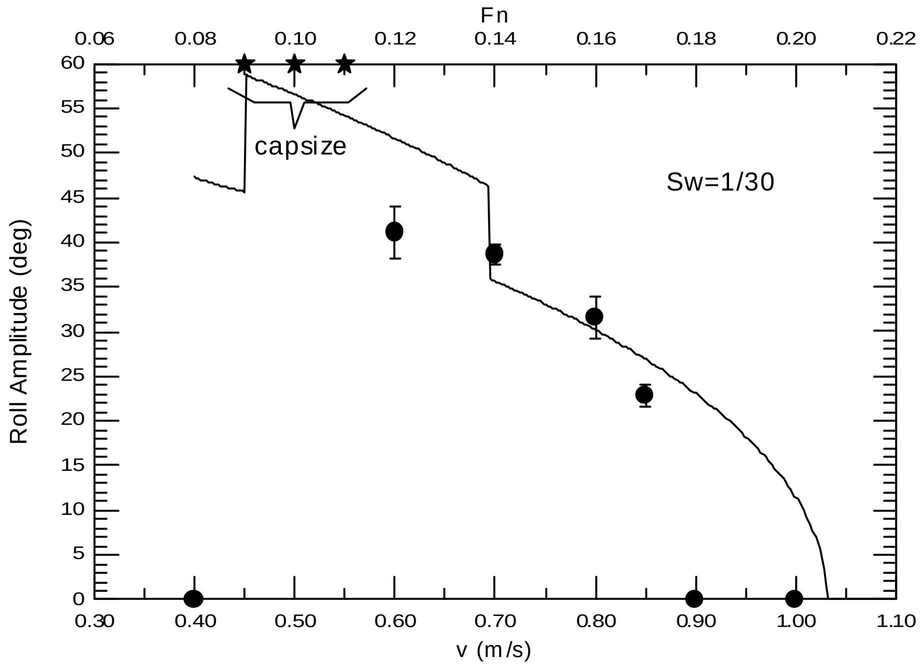

Fig. 6. Experimental results versus simulation based on Eq. 16-17 for the case s_w=1/30.

The comparison is satisfactory. It is worth noting the case sw=1/30, where the jump in the simulated amplitude is close to the zone where the experiments gave very large amplitudes with capsizing tendency (the model was restrained not to reach excessive roll amplitudes). This jump is tied to the fact that p_2 crosses zero at an inclination of about 40 deg, *and then recovers* due to the inversion of the curves. This behaviour, although resulting from an analysis based on some strong assumptions, is nevettheless puzzling.

The estimated parameter values are in qualitative agreement and follow the trend given by hydrostatic calculations, but generally the good simulation was obtained with higher values of α_3 and lower values of p_1.

This can be due to the vertical motions of the ship and to the partial equivalency of the terms corresponding to the two factors in the perturbative approach.

The comparison with the full Eq. 16 and with the coupled system Eq. 19 is in progress due to the need to introduce in the Parameter Identification Technique the solution of the above systems of equations.

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