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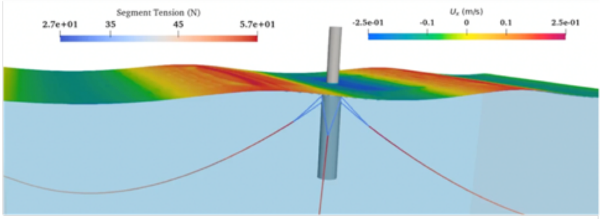
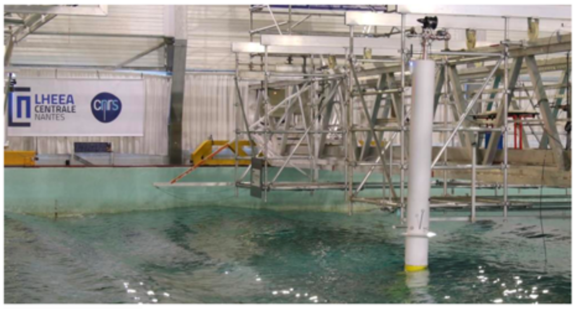



# COMPARAISON DES MODÈLES DE *STRETCHING* ET D'UNE THÉORIE DE PROPAGATION NON-LINÉAIRE DE VAGUES POUR LE CALCUL DE LA CINÉMATIQUE DANS LES CRÊTES

# *COMPARISON OF STRETCHING MODELS WITH A FULLY NONLINEAR WAVE THEORY IN THE COMPUTATION OF THE KINEMATICS OF WAVE CRESTS*

**P.-Y. WUILLAUME[(1)], G. DE HAUTECLOCQUE[(2)], I. TEN[(2)]**
*pierre-yves.wuillaume@bureauveritas.com ; guillaume.de-hauteclocque@bureauveritas.com*
*igor.ten@bureauveritas.com*

[(1)]Département Recherche, Bureau Veritas, Nantes, France
[(2)]Département Recherche, Bureau Veritas, Paris, France

**Résumé**

L'équation de Morison est fréquemment utilisée lors de la conception des structures flottantes. Elle repose sur la connaissance de la cinématique des vagues. Du fait de l'hypothèse de faible cambrure des vagues, les modèles de vague premier et deuxième ordre ne peuvent prédire la cinématique au-dessus de la surface libre moyenne jusque dans les crêtes. Pour pallier cette difficulté, des modèles de *stretching* sont généralement utilisés. D'autre part, des modèles de vagues non-linéaires sont aujourd'hui disponibles pour estimer avec précision la cinématique des vagues cambrées. Si ces modèles sont trop couteux pour la plupart des applications industrielles, ils fournissent des solutions de référence intéressantes. Dans cette étude, les cinématiques de houle prédites par les différents modèles de *stretching* ont été évaluées sur houle régulière et irrégulière, en profondeur infinie et sans déferlement. Nous montrons que le modèle de Wheeler est à éviter (sous-estimation importante des vitesses), et que les modèles second-ordre et *delta-stretching* fournissent de bonnes approximations.

**Summary**

The Morison's equation is often used in the design of floating structues. It relies on the knowledge of the wave kinematics. Due to the small amplitude assumption, the first and second order wave models can not directly predict the kinematics above the mean sea

level up to the wave crest. To overcome this issue, simple stretching models are generally used. On the other hand, fully nonlinear wave models are now able to estimate with accuracy the kinematics, even for steep waves. Those nonlinear models are too CPU intensive for most of industrial applications, but provide a valuable reference solution. In this study, the different stretching models are thus evaluated, on regular and irregular non-breaking waves, in infinite water-depth. It is shown that the wheeler model should be avoided (large under-estimation of velocities), and that second-order and delta-stretching approaches provide good approximations.

## I – Introduction

The Morison's equation [9] is frequently used to model the hydrodynamic loads of mooring lines or braces of floating offshore wind turbines. It requires the knowledge of the wave kinematics up to the wave crests. The wave velocity obtained with the linear theory is divergent above the mean water level which involves large errors in the evaluation of the Morison's equation, in particular in the drag term proportional to the velocity squared. The reason is that, in the linear theory, free surface boundary conditions are linearized around the mean water level ($z = 0$), which becomes the reference level of both long and small waves. However, physically, small waves overlap long waves [8]. In practice, this difficulty is overcome by using approximated models, named *stretching models*, which evaluate the wave velocity above the mean water lever using the information below $z = 0$.

Stretching models have started with the linear extrapolation, which linearly extrapolates the kinematics up to the wave crest. Then the Wheeler model has been derived [13] which maps the vertical scale to evaluate the kinematics at the wave crest from its value at the mean water level. Having shown that the linear extrapolation and the Wheeler stretching provide the upper and lower bounds, respectively, of observed data, the delta-stretching model has been introduced by [10] to mixe the two previous models. More recently, a second-order stretching model has been studied by [11]. Using PIV measurements, the second-order model was considered as better than the Wheeler model.

Nowadays, nonlinear wave propagation solvers such as HOS (High-Order Spectral) are widely accessible [3], providing a valuable reference to benchmark simplified stretching models. The goal of this study is to compare the wave velocity above the mean sea level from the different stretching models and using a nonlinear wave model (HOS or stream function theory [5]) as a reference, both for regular and irregular waves. It is assumed that waves are non-breaking and the water depth is infinite.

This paper begins with the presentation of the wave models and the associated stretching models. Then a comparison is performed on regular waves and irregular design wave.

## II – Wave models

A Cartesian system of coordinate $(x, y, z)$ is defined such as the plane $(xOy)$ lies on the mean water surface and the axis $(Oz)$ points upward. The water depth is assumed infinite. The flow is considered as irrotational and incompressible and the fluid inviscid, which leads to use the potential flow theory such as the fluid velocity derives from a velocity potential:

$$\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \nabla\phi \tag{1}$$

### II – 1 First-order wave

Using the small wave steepness assumption, the first-order velocity potential in infinite water depth satisfies:

$$\begin{cases} \Delta\phi^{(1)} = 0 \text{ in the fluid domain} \\ \phi^{(1)}_{tt} + g\phi^{(1)}_{z} = 0 \text{ for } z = 0 \\ \|\nabla\phi^{(1)}\| \to 0 \text{ for } z \to -\infty \end{cases} \tag{2}$$

The solution for a wave propagating along $(Ox)$ is:

$$\phi^{(1)}(x,z,t) = \frac{Ag}{\omega} e^{kz} \sin(kx - \omega t) \tag{3}$$

with $A$ the wave amplitude, $g$ the gravity constant, $\omega$ the wave frequency and $k = \omega^2/g$ the wave number. The corresponding wave elevation is expressed by:

$$\eta^{(1)}(x,t) = A\cos(kx - \omega t) \tag{4}$$

And the horizontal velocity arises:

$$u(x,z,t) = A\omega e^{kz} \cos(kx - \omega t) \tag{5}$$

This expression is divergent for $z > 0$. The superposition principle gives the velocity potential for an irregular sea state:

$$\phi^{(1)}(x,z,t) = \sum_{n=1}^{N} \frac{A_n g}{\omega_n} e^{k_n z} \sin(k_n x - \omega_n t + \theta_n) \tag{6}$$

In the following sections, the stretching associated with the first-order wave model are detailed.

### II – 1.1 Linear extrapolation model

A first basic model expresses the wave velocity as a Taylor series to first order about the point $z = 0$ [8]:

$$u(x,y,z,t) = u^{(1)}(x,y,0,t) + z\frac{\partial u^{(1)}}{\partial z}(x,y,0,t) \text{ for } z \in \left[0, \eta^{(1)}\right] \tag{7}$$

### II – 1.2 Wheeler model

The Wheeler model [13] stretches the vertical axis from $[-h, \eta^{(1)}]$ to $[-h, 0]$ with $h$ the water depth, leading to:

$$u(x,y,z,t) = u^{(1)}(x,y,z_{Wheeler},t) \text{ for } z \in \left[0, \eta^{(1)}\right] \tag{8}$$

with:

$$z_{Wheeler} = h\frac{z - \eta^{(1)}}{h + \eta^{(1)}} \tag{9}$$

In infinite water depth, the vertical position is simply:

$$z_{Wheeler} = z - \eta^{(1)} \tag{10}$$

### II – 1.3 Delta-stretching model

The delta-stretching model makes the transition between the linear extrapolation and the Wheeler model [10]. This model is based on two parameters: $\Delta \in [0,1]$ and $h_\Delta$ which represents the water depth from which the stretching is applied. The vertical axis is stretched from $[-h_\Delta, \eta^{(1)}]$ to $[-h_\Delta, \Delta\eta^{(1)}]$. The wave velocity is expressed by:

$$u(x,y,z,t) = \begin{cases} u^{(1)}(x,y,0,t) + z_\Delta \frac{\partial u^{(1)}}{\partial z}(x,y,0,t) \text{ for } z_\Delta \in \left[0, \Delta\eta^{(1)}\right] \\ u^{(1)}(x,y,z_\Delta,t) \text{ for } z_\Delta < 0 \end{cases} \tag{11}$$

with:

$$z_\Delta = (z + h_\Delta)\frac{h_\Delta + \Delta\eta^{(1)}}{h_\Delta + \eta^{(1)}} - h_\Delta \tag{12}$$

In infinite water depth, this expression becomes:

$$z_\Delta = z + (\Delta - 1)\eta^{(1)} \tag{13}$$

The delta-stretching model matches the linear extrapolation for $(\Delta, h_\Delta) = (1, h)$ and the Wheeler model for $(\Delta, h_\Delta) = (0, h)$. In practice, these parameters are set to: $(\Delta, h_\Delta) = (0.3, h)$ [10, 8]

### II − 2 Second-order wave

The second-order velocity potential in infinite water depth satisfies:

$$\begin{cases} \Delta\phi^{(2)} = 0 \text{ in the fluid domain} \\ \phi_{tt}^{(2)} + g\phi_z^{(2)} = Q \text{ for } z = 0 \\ \|\nabla\phi^{(2)}\| \to 0 \text{ for } z \to -\infty \end{cases} \tag{14}$$

where the right-hand side of the free surface boundary condition is defined as:

$$Q = -\eta^{(1)}\left(\phi_{ttz}^{(1)} + g\phi_{zz}^{(1)}\right) - 2\nabla\phi^{(1)} \cdot \nabla\phi^{(1)} \tag{15}$$

Its general form for a irregular sea state is:

$$Q = \Re\left[\sum_{n=1}^{N}\sum_{m=1}^{N}\left(q_{nm}^{+}e^{-i(\omega_n+\omega_m)t} + q_{nm}^{-}e^{-i(\omega_n-\omega_m)t}\right)\right] \tag{16}$$

The solution of the boundary value problem is expressed as the sum of two velocity potentials: a sum-frequency velocity potential and a difference-frequency velocity potential:

$$\phi^{(2)} = \phi^{(2+)} + \phi^{(2-)} \tag{17}$$

$$= \Re\left[\sum_{n=1}^{N}\sum_{m=1}^{N}\left(\varphi_{nm}^{(2+)} + \varphi_{nm}^{(2-)}\right)\right] \tag{18}$$

with:

$$\varphi_{nm}^{(2)\pm} = A_n A_m i\alpha_{nm}^{+} e^{k_{nm}^{\pm}z}\frac{e^{i(k_{nm}^{\pm}x - \omega_{nm}^{\pm}t)}}{gk_{nm}^{\pm} - (\omega_{nm}^{\pm})^2} \tag{19}$$

with $A_n$ the incident wave amplitude, $\omega_{nm}^{\pm} = \omega_n \pm \omega_m$, $k_{nm}^{\pm} = k_n \pm k_m$ and $\alpha_{nm}$ the bichromatic wave amplitude due the interaction between the $n$-th and $m$-th waves:

$$\begin{cases} \alpha_{nm}^{-} = 2\omega_n\omega_m\omega_{nm}^{-} \\ \alpha_{nm}^{+} = 0 \end{cases} \tag{20}$$

The horizontal second-order wave velocity is defined by:

$$u^{(2)} = \frac{\partial\phi^{(2)}}{\partial x} \tag{21}$$

The second-order wave elevation is expressed by:

$$\eta^{(2)} = -\frac{1}{g}\left[\eta^{(1)}\phi_{zt}^{(1)} + \frac{1}{2}\left(\nabla\phi^{(1)}\right)^2 - \phi_t^{(2)} \text{ for } z = 0\right] \tag{22}$$

The wave crest is reached at $z = \eta^{(1)} + \eta^{(2)}$, contrary to the first-order model where it is at $z = \eta^{(1)}$.

The second-order stretching model uses the linear extrapolation enhanced by the second-order term [11]:

$$u(x, y, z, t) = u^{(1)}(x, y, 0, t) + z\frac{\partial u^{(1)}}{\partial z}(x, y, 0, t) + u^{(2)}(x, y, 0, t) \text{ for } z \in \left[0, \eta^{(1)} + \eta^{(2)}\right] \tag{23}$$

### II – 3 Fully non-linear models

The reference models used to benchmark are fully non-linear: the stream function theory (detailed in [4]) for regular waves, HOS-NWT (detailed in [2]) for irregular waves.

## III – Results

### III – 1 Regular waves

The aims is, for a given regular wave, to compare the horizontal velocity profile provided by the different models. However the wave height, amplitude, period and length cannot be all made identical in all models. Firstly either amplitude or height, then, either wave period or wave length can only match. In the following, we have chosen to match the wave crest; the velocity profile at the wave crest is studied in two cases:

- the wave period $T$ and the wave crest $A_c$ are prescribed ; the wave steepness is defined by:

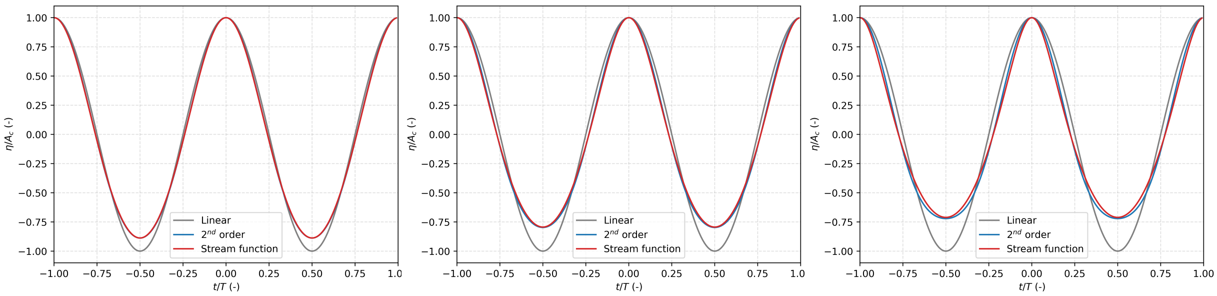
$$\epsilon_T = \frac{4\pi A_c}{gT^2} \tag{24}$$

- the wave length $\lambda$ and the wave crest $A_c$ are prescribed ; the wave steepness is defined by:

$$\epsilon_\lambda = \frac{2A_c}{\lambda} \tag{25}$$

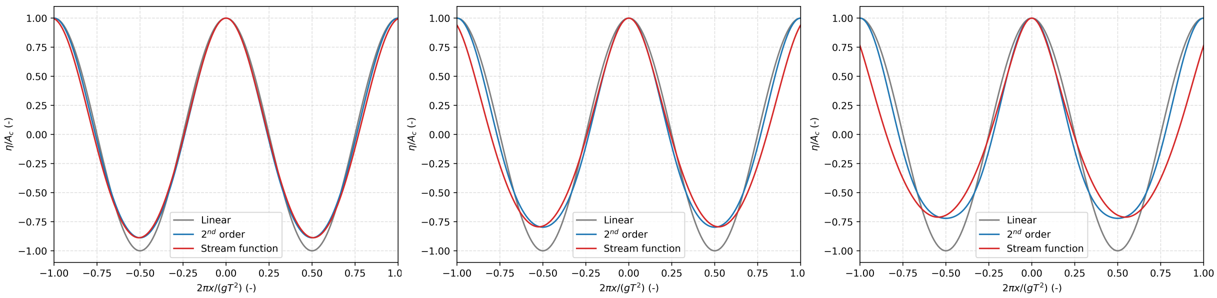
Simulations have been performed for $\epsilon_T \in [0.04, 0.12]$ and $\epsilon_\lambda \in [0.04, 0.1]$.

Regular wave elevations are presented for $\epsilon_T = 0.04$, 0.08 and 0.12 in Fig. 1 with respect to time and in Fig. 2 with respect to the horizontal position. As the wave steepness increases, differences between the wave models arise due to the non-linearities. In particular, the wave length for the stream function wave increases. The first-order wave stays unchanged. The horizontal wave velocity profiles at the wave crest location ($x = 0$ and $t = 0$) obtained with each stretching model and the stream function theory are presented in Fig. 3 for $\epsilon_T = 0.04$, 0.08 and 0.12. The Wheeler model underestimates the horizontal velocity while the linear extrapolation overestimates it. From their definition, the Wheeler model gives the same velocity at $z = A_c$ and $z = 0$ respectively than the linear extrapolation model at $z = 0$ and $z = -A_c$, respectively. For each wave steepness, the horizontal velocity based on the stream function is always between the delta stretching model and the second-order wave model.



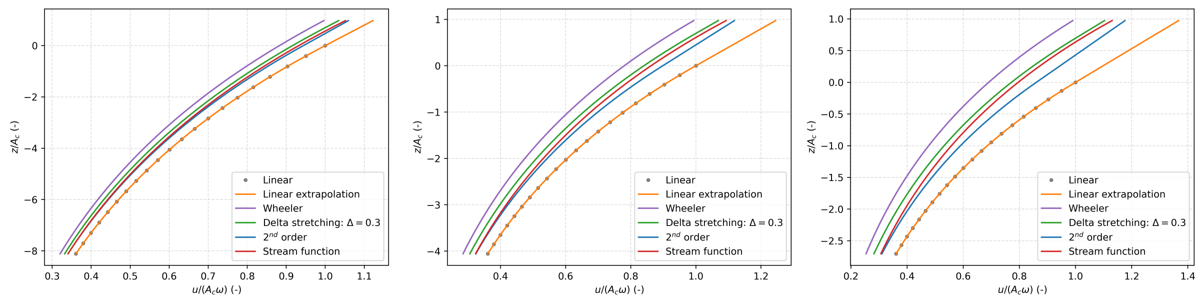

(a) $\epsilon_T = 0.04$ (b) $\epsilon_T = 0.08$ (c) $\epsilon_T = 0.12$

Figure 1: Regular wave elevation with respect to the time for the first (grey) and second (blue) order waves and the stream function wave (red) for $\epsilon_T = 0.04$, 0.08 and 0.12




(a) $\epsilon_T = 0.04$ (b) $\epsilon_T = 0.08$ (c) $\epsilon_T = 0.12$

Figure 2: Regular wave elevation with respect to the horizontal position for the first (grey) and second (blue) order waves and the stream function wave (red) for $\epsilon_T = 0.04$, 0.08 and 0.12




(a) $\epsilon_T = 0.04$ (b) $\epsilon_T = 0.08$ (c) $\epsilon_T = 0.12$

Figure 3: Horizontal velocity profile at the wave crest location with respect to the vertical position each stretching model and the stream function wave for $\epsilon_T = 0.04$, 0.08 and 0.12

These results can also be seen if the horizontal velocity at the wave crest ($z = A_c$) is displayed with respect to the wave steepness (Fig. 4). The differences between the stretching models grow with the wave steepness. But it can be stated that for every wave steepness, the second-order model and the delta stretching give the better results. Nevertheless, this latter model is based on the parameter $\Delta \in [0, 1]$. A sensitivity analysis has been performed for the highest value of $\epsilon_T$ to study the effect of this parameter and

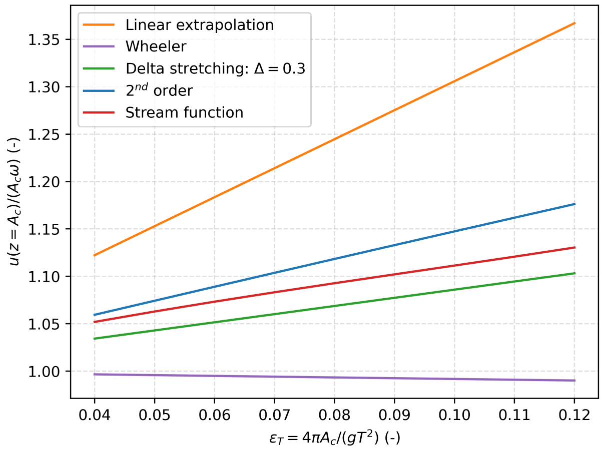



Figure 4: Horizontal velocity at the wave crest with respect to the wave steepness $\epsilon_T$

is shown in Fig. 5. Intermediate values of $\Delta$ are the most consistent with the stream function results.

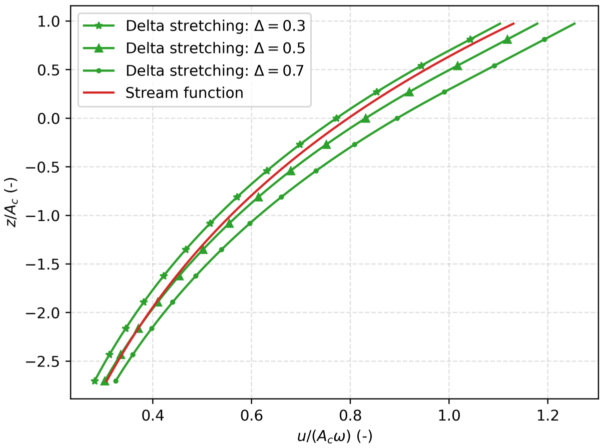



Figure 5: Sensitivity of the delta-stretching model to the $\Delta$ parameter for $\epsilon_T = 0.12$

The second case where the wave length $\lambda$ is prescribed gives similar trends (Fig. 6).

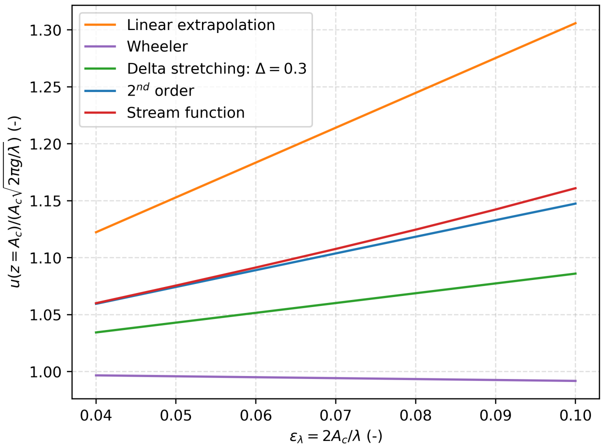



Figure 6: Horizontal velocity at the wave crest with respect to the wave steepness $\epsilon_\lambda$

## III – 2 Irregular waves

Irregular sea states are now considered. As waves do not propagate in the same way in the first and second order models and in HOS, comparing the kinematics at a given instant is not trivial. To overcome this issue, a design wave approach is used to provide comparable results between models. The design wave is here defined as the most probable wave which reaches a specified wave crest for a given wave spectrum. For the linear case, the design wave can be obtained analytically [12]. For the nonlinear cases (second-order and HOS), calculating the design wave involves a minimization under constraint (the reliability index is to be minimized under the constraint that the crest reach the desired target); implementation details can be found in [7].

Design waves have been generated for wave steepness range of $\epsilon_{T_p} \in [0.02, 0.04, 0.06, 0.08]$ with:

$$\epsilon_{T_p} = \frac{4\pi A_c}{g T_p^2} \tag{26}$$

The spectrum shape considered is a Jonswap with $\gamma = 1.5$ [6]. For the linear model, it can be shown that the design wave does not depend on the significant wave height [1], which is not garanteed in the nonlinear case. As a check. Figure 7 shows a design wave calculated for $\epsilon_{T_p} = 0.02$ for different significant wave heights ($A_c = 3H_s/5$ and $A_c = H_s$). The two waves are considered very close. In the remaining comparisons, $A_c = H_s$ will be used.

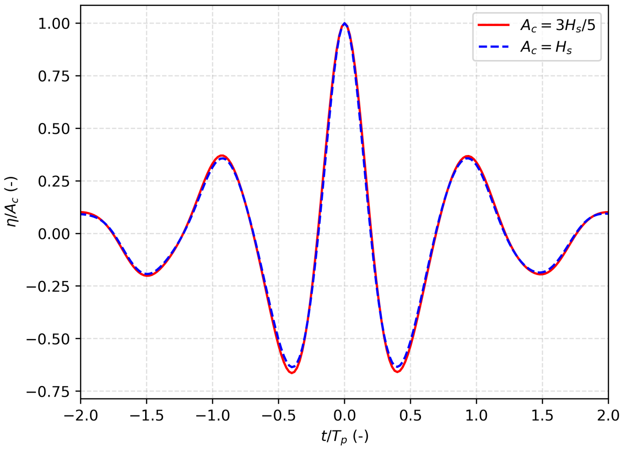
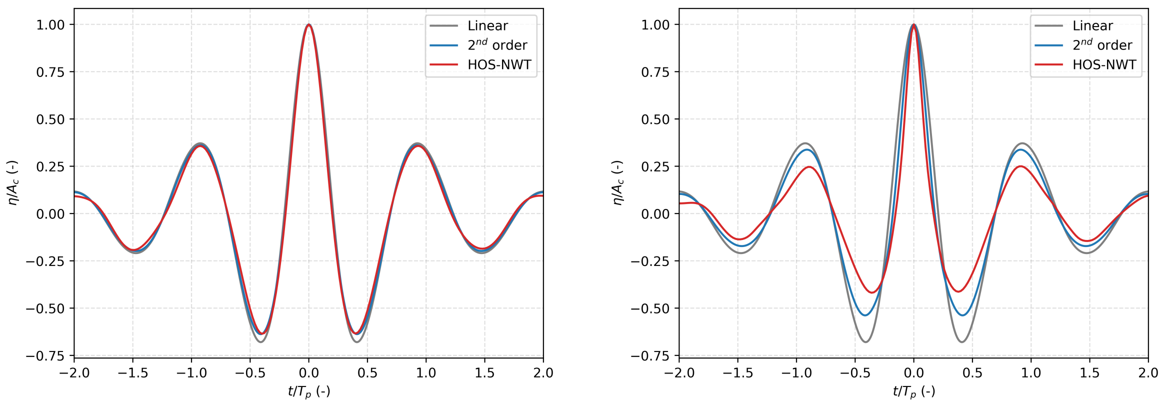



Figure 7: Design waves with respect to the time using HOS for $\epsilon_{T_p} = 0.02$

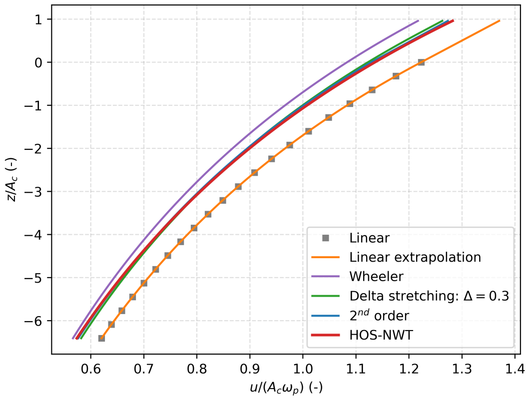
The design waves for $\epsilon_{T_p} = 0.02$ and 0.08 are presented in Fig. 8. As the wave steepness increases, differences between the wave models arise due to the non linearities.



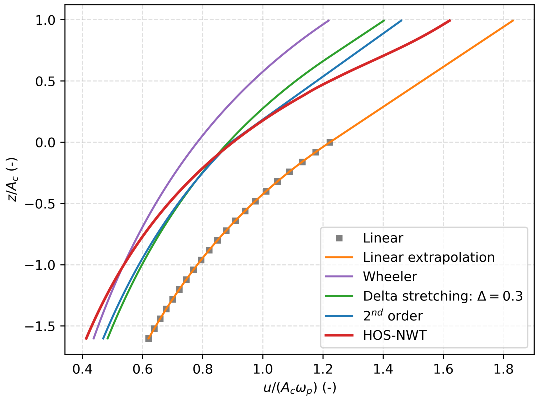

(a) $\epsilon_{T_p} = 0.02$ (b) $\epsilon_{T_p} = 0.08$

Figure 8: Design waves with respect to the time for the first (grey) and second (blue) order waves and HOS (red) for $\epsilon_{T_p} = 0.02$ and 0.08

The horizontal wave velocity profiles at the wave crest location ($x = 0$ and $t = 0$) obtained with each stretching model and the stream function theory are presented in Fig. 9 for $\epsilon_{T_p} = 0.02$, and 0.08. As for the regular wave case, the Wheeler model underestimates the horizontal velocity while the linear extrapolation overestimates it as expected. For each wave steepness, the horizontal velocity evaluated from the second-order wave model is the closest to on HOS. This observation can also be stated if the horizontal velocity at the wave crest ($z = A_c$) is displayed with respect to the wave steepness (Fig. 10). A sensitivity analysis about $\Delta$ has been performed for the delta-stretching model with $\epsilon_{T_p} = 0.08$ and is shown in Fig. 11. Low values of $\Delta$ are the most consistent close to $z = 0$, higher values are preferable in the wave crest.




(a) $\epsilon_{T_p} = 0.02$




(b) $\epsilon_{T_p} = 0.06$

Figure 9: Horizontal velocity profile at the wave crest location with respect to the vertical position each stretching model and HOS for $\epsilon_T = 0.02$ and 0.08

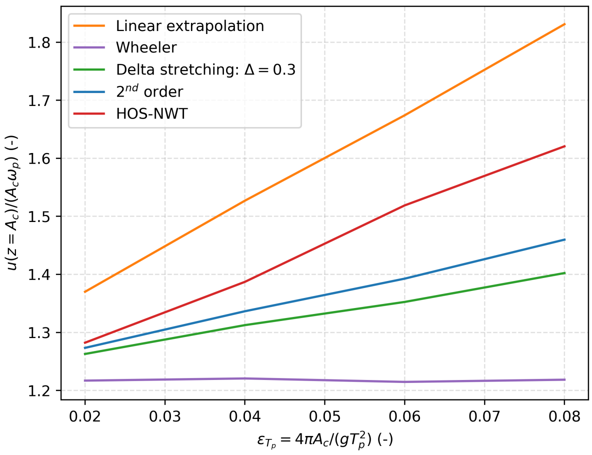



Figure 10: Horizontal velocity at the wave crest with respect to the wave steepness $\epsilon_{T_p}$

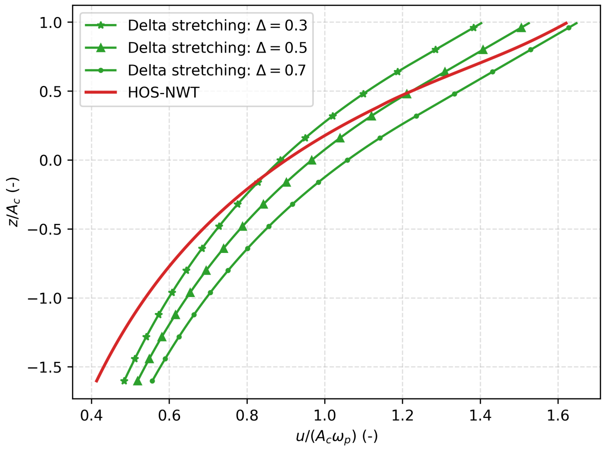



Figure 11: Sensitivity of the delta-stretching model to the $\Delta$ parameter for $\epsilon_{T_p} = 0.08$

# IV – Conclusions

A comparative study was conducted to benchmark the performance of various stretching models to evaluate wave kinematic under the crest. On regular waves, the stream function theory was used as reference. The linear extrapolation overestimates the wave velocity while the Wheeler model underestimates it, as expected. The delta-stretching model for $\Delta = 0.3$ and the second-order model give decent estimations of the wave velocity. On irregular waves, a design wave approach and the HOS wave model for reference have been chosen. Similar trends in the results have been observed.

Overall, the second-order model may be considered as the most robust stretching model, even if, in steep sea states, differences compared to a nonlinear wave theory still occur. The second-order model is relatively cheap compared to HOS, but still more expensive than the delta-stretching, which is thus a good alternative (with $\Delta = 0.3$). On the other hand, Wheeler stretching should be avoided as it significantly underestimates the velocity.

In this work, we have compared the various models using a given crest height, which is a debatable approach. Indeed, for a given sea-state, the extreme crests (at a given return period) are not the same for all models. The conclusions here drawn on velocity profile might thus not be directly transferable to the extreme loads on a structure. For instance, the over-estimation of the velocity by the linear extrapolation method will be compensate (probably partially or too much) by the under-estimation of the linear crests. A possible extension of this work could, for instance, compare velocity profiles on irregular waves targeting a crest height at a given return period (keeping the same return period for all models, the targeted crest will be different).

More pragmatically, the effect of the different stretching models on the extremes of Morison's loads on a simple vertical cylinder would be interesting to investigate. And even better, the effect on the final response of actual floating offshore wind turbines (using a time-domain solver) could be estimated and allow to conclude on the best CPU-time/accuracy compromises in industrial cases.

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