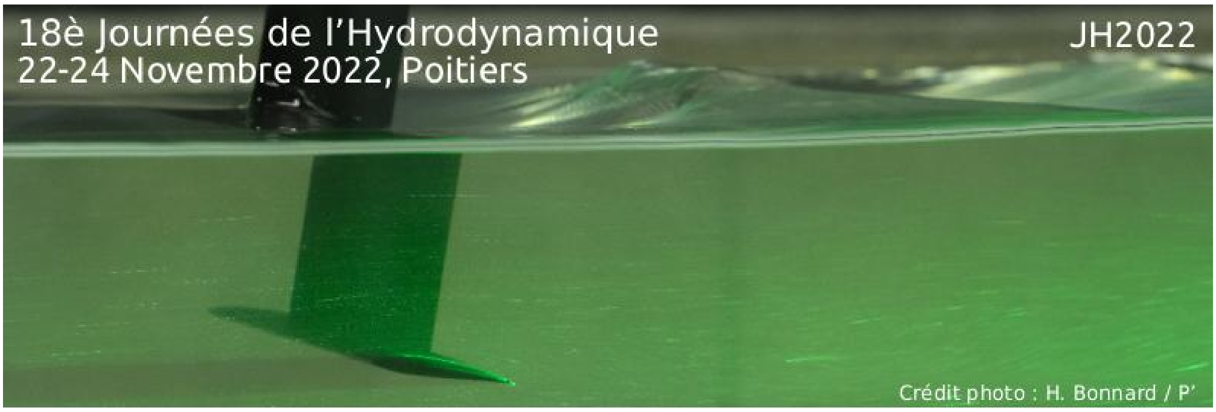

FORMULATIONS AUX CHAMPS PROCHES, INTERMEDIAIRES ET LOINTAINS POUR LA RESISTANCE AJOUTEE

NEARFIELD, MIDFIELD AND FARFIELD FORMULATIONS OF ADDED RESISTANCES

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Résumé

La resistance ajoutée d'un navire s'avançant dans la houle est formulée par trois approches distinguées. La formulation aux champs proches est obtenue par l'intégration de la pression du second ordre et de la pression linéaire sur la surface intermittente ainsi que des variations des efforts linéaires dues aux mouvements. La formulation aux champs intermédiaires est donnée par le principe de conservation des quantités de mouvement dans un domaine limité par une surface de contrôle entourant la carène mais à une distance finie. Enfin, la formulation aux champs lointain est déduite en considérant une surface de contrôle se situant à l'infinie sur laquelle les expressions asymptotiques du potentiel de perturbation sont utilisées. Ces trois formulations sont développées afin de clarifier les controverses dans les études précédentes et de les enlever autant que possible. Des analyses sont effectuées sur les formulations pour identifier leur avantages et inconvénients dans les applications.

Summary

The added-resistance of ship advancing in waves is formulated in three different manners. The classical nearfield formulation is resultant of pressure integrations on the ship hull. By considering a fluid domain limited by a control surface surrounding the ship but at a finite distance, the midfield formulation is obtained based on the conservation principle of momentum in the domain. The farfield formulation is derived by putting the control surface at infinity on which asymptotic expressions of perturbed potentials are used. These formulations are developed in order to clarify and remove the controversy in previous studies. Some analyses are summarized to identify their advantages and drawbacks, and missing terms.

I – Introduction

A novel method based on the linearization over the ship-shaped stream (double-body flow) and the use of free-surface Green function with viscosity has recently been developed [2] to compute wave loads on a ship advancing in waves and induced motions. Perfect agreement has been achieved between the numerical results and benchmark model tests concerning the added mass, radiation damping, wave excitation loads and ship motions. Being critically important, here we consider the second-order mean forces applied on the ship, in particular, the added-resistance due to unsteady flows around the ship advancing in waves.

The definition of different reference systems used in the analysis and the representation of rotations are presented in Section II, together with some basic notations of perturbation analysis. The nearfield formulation has been developed by many previous studies for zero-speed case. The analysis given in [8] and [6] is followed and extended to the forward-speed case, in Section III. By considering a finite domain surrounding the ship by a control surface at a finite distance as in [1], the application of the conservation principle of momentum included in the domain yields the midfield formulation, in Section IV, to compute the second-order mean forces by only the integral on the control surface and along its intersection with the mean free surface. The farfield formulation can be obtained by putting the control surface to the infinity at which asymptotic expressions of wave fields can be used. The farfield formulation by Kashiwagi given in [5] is reported in Section V. Finally, some discussions and conclusions are given in Section VI.

II – Ship kinematic representations

We start with the definition of several reference systems used to describe ship's motions and fluid flows around the ship advancing in waves. The axis-angle representation of rotations by Rodrigues is given and the waterline integral involved in the second-order load formulations is then analysed.

II – 1 Reference systems

Considering the ship advancing along a straight path with a constant speed U, the **straight-forwarding system** $O(X,Y,Z)$ going with the same speed U, and along the same straight path is usually defined with the origin located on the mean free surface, and by its OX-axis oriented forward (bow), OY-axis pointing port-side and OZ-axis positive upwards. An **earth-fixed system** is classically needed to describe the straight-forwarding system with absolute coordinates. Here, we just mention it since it is not used otherwise. The straight-forwarding reference system is an inertial system so that all physical laws valid in the earth-fixed system stay valid in $O(X,Y,Z)$. The ship forward speed is then represented by the uniform stream in the negative OX-axis direction. In the vicinity of the ship, there are, in addition, the ship-shaped stream and wavy steady flow, and unsteady flow due to wave diffraction and radiation.

We define the **ship-fixed system** by attaching it with the ship. Often denoted by $o(x,y,z)$, it is used to describe the ship geometry and inertial properties such as center of buoyancy (CoB), center of gravity (CoG), inertial radii, etc. The ship-fixed system is, indeed, not an inertial system, if ship motions are unsteady. It can be coincided with the straight-forwarding system in the static or/and steady situations. In the unsteady time-harmonic motions, the vectors in the ship-fixed reference system are at their temporal mean positions.

Finally, a so-called **translation system** denoted by $o'(x',y',z')$ is defined to be in translation motions. It is in parallel with the straight-forwarding system $O(X,Y,Z,t)$ but different by the translations of the ship, i.e.,

$$(X,Y,Z) = (X_o,Y_o,Z_o) + (x',y',z') + (\xi_1,\xi_2,\xi_3) \tag{1}$$

in which (X_o,Y_o,Z_o) are the coordinates of the origin o' of the translation system and (ξ_1,ξ_2,ξ_3) the translations in surge, sway and heave, respectively. Since the origins o and o' are coincided, the difference between the ship-fixed system $o(x,y,z)$ and the translation system $o'(x',y',z')$ is only due to the rotations $(\theta_1,\theta_2,\theta_3)$ in roll, pitch and yaw around o (or o'), respectively. Strictly speaking, this translation system is not necessary to quantify actions (loads) and induced motions of ship. It can be done directly in the straight-forwarding system by using the geometrical

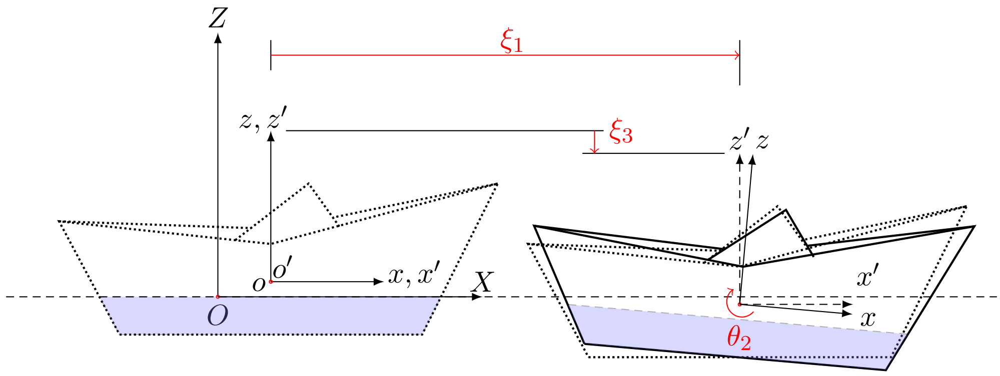


FIGURE 1 – Reference systems and ship motions

description in the ship-fixed system. On the other side, the translation system can be very useful in the development of different formulations, as did in [8] and in [6].

Above reference systems are illustrated on Figure 1 with help of a paper boat. On the left, the straight-forwarding system and ship-fixed system which coincides the translation system, are in parallel when the ship is advancing steadily in calm water. Under the oscillatory action of waves, the ship oscillates around its mean position. The translations change the position of ship-fixed system which coincides with the translation system. On the right, the ship rotation around the origin changes the ship-fixed system with respect to the translation system.

II – 2 Rotation representation

The angle-axis representation is adopted here to represent ship rotation motions. As explained in [6], the rotation angles around three axes (roll, pitch and yaw) of unit vectors $(\mathbf{e}_1, \mathbf{e}_3, \mathbf{e}_3)$ denoted by the vector $\boldsymbol{\theta} = \theta_1\mathbf{e}_1 + \theta_2\mathbf{e}_3 + \theta_3\mathbf{e}_3 = (\theta_1, \theta_2, \theta_3)$ can be realised once by the angle θ around the unit vector $\hat{\boldsymbol{n}}$ which are given by

$$\theta = |\boldsymbol{\theta}| = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2} \quad \text{and} \quad \hat{\boldsymbol{n}} = \boldsymbol{\theta}/\theta \tag{2}$$

Any vector $\boldsymbol{r} = (x, y, z)$ with its three components in the translation system equal to

$$\boldsymbol{r} = \hat{\boldsymbol{n}}(\hat{\boldsymbol{n}} \cdot \boldsymbol{r}) + (\hat{\boldsymbol{n}} \wedge \boldsymbol{r}) \wedge \hat{\boldsymbol{n}} \tag{3}$$

becomes

$$\boldsymbol{r}' = (x', y', z') = \hat{\boldsymbol{n}}(\hat{\boldsymbol{n}} \cdot \boldsymbol{r}) + \cos\theta(\hat{\boldsymbol{n}} \wedge \boldsymbol{r}) \wedge \hat{\boldsymbol{n}} + \sin\theta(\hat{\boldsymbol{n}} \wedge \boldsymbol{r}) \tag{4}$$

after the rotation θ around the axis $\hat{\boldsymbol{n}}$, as illustrated on the left of Figure 2. By using the identity $(\hat{\boldsymbol{n}} \wedge \boldsymbol{r}) \wedge \hat{\boldsymbol{n}} = \boldsymbol{r} - \hat{\boldsymbol{n}}(\hat{\boldsymbol{n}} \cdot \boldsymbol{r})$ and introducing (2) in (4), we have

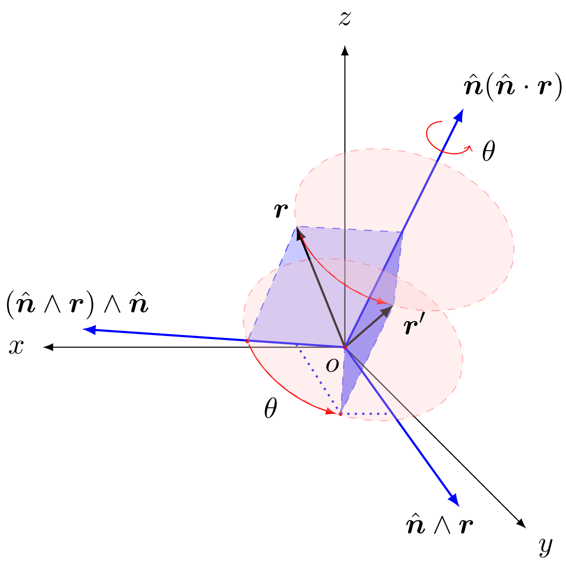
$$\boldsymbol{r}' = \mathbb{T}\boldsymbol{r} \quad \text{with the transfer matrix} \quad \mathbb{T} = \mathbb{I} + \frac{\sin\theta}{\theta}\mathbb{T}_1 + \frac{1-\cos\theta}{\theta^2/2}\mathbb{T}_2 \tag{5}$$

in which $\mathbb{I}$ is the unit matrix, $\mathbb{T}_1$ is a skew-symmetric matrix given by

$$\mathbb{T}_1 = \begin{bmatrix} 0, & -\theta_3, & \theta_2 \\ \theta_3, & 0, & -\theta_1 \\ -\theta_2, & \theta_1, & 0 \end{bmatrix} \quad \text{with application} \quad \mathbb{T}_1(\boldsymbol{\theta})\boldsymbol{r} = \boldsymbol{\theta} \wedge \boldsymbol{r} \tag{6}$$

and the symmetric matrix $\mathbb{T}_2$ given by

$$\mathbb{T}_2 = \frac{1}{2}\begin{bmatrix} -\theta_2^2 - \theta_3^2, & \theta_1\theta_2, & \theta_1\theta_3 \\ \theta_2\theta_1, & -\theta_3^2 - \theta_1^2, & \theta_2\theta_3 \\ \theta_3\theta_1, & \theta_3\theta_2, & -\theta_1^2 - \theta_2^2 \end{bmatrix} \quad \text{with application} \quad \mathbb{T}_2(\boldsymbol{\theta})\boldsymbol{r} = \frac{1}{2}\boldsymbol{\theta} \wedge (\boldsymbol{\theta} \wedge \boldsymbol{r}) \tag{7}$$



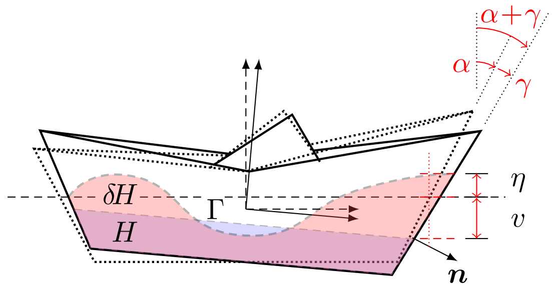


FIGURE 2 – Rotation by the axis-angle representation of Rodrigues (left) and intermittent surface δH around the waterline Γ (right)

It can be shown that

$$\mathbb{T}^{-1} = \mathbb{I} - \frac{\sin\theta}{\theta}\mathbb{T}_1 + \frac{1-\cos\theta}{\theta^2/2}\mathbb{T}_2 = \mathbb{T}^{\mathsf{t}} \tag{8}$$

with $\mathbb{T}^{\mathsf{t}}$ is the transpose of $\mathbb{T}$. Furthermore, we have

$$|\mathbb{T}| \equiv 1\,; \quad \mathbb{T}_1\mathbb{T}_1 = 2\mathbb{T}_2 = \mathbb{T}_1^{\mathsf{t}}\mathbb{T}_1^{\mathsf{t}} \quad \text{and} \quad \mathbb{T}_1^{\mathsf{t}}\mathbb{T}_1 = -2\mathbb{T}_2 = \mathbb{T}_1\mathbb{T}_1^{\mathsf{t}} \tag{9}$$

which are useful in the following.

II – 3 Integrations on the instantaneous hull

The wetted part of ship hull is varying in time due to its oscillatory motions and wave elevations around the waterline. The instantaneous ship hull $\mathcal{H}$ can be considered as the sum of the instantaneous hull under the waterline H and the intermittent surface δH around the mean waterline Γ, defined as the difference between $\mathcal{H}$ and H, i.e., $\mathcal{H} = H \cup \delta H$. The integral of any function $f(X,Y,Z)$ on the instantaneous hull $\mathcal{H}$ can be evaluated by

$$\iint_{\mathcal{H}} f(X,Y,Z,t)\,\mathrm{d}S = \iint_{H} f(X,Y,Z,t)\,\mathrm{d}S + \iint_{\delta H} f(X,Y,Z,t)\,\mathrm{d}S \tag{10}$$

These integrals can be written equivalently

$$\iint_{H} f(X,Y,Z,t)\,\mathrm{d}S = \iint_{H} f(x',y',z',t)\,\mathrm{d}s \tag{11}$$

in the translation coordinate system. Considering the hull piercing the mean free surface straightly with an angle α with the vertical line, i.e., $\alpha = 0$ if the hull is wall-sided. Along the waterline Γ, the wave elevation is denoted by η and its vertical displacement by v, the integral on the intermittent surface is then

$$\iint_{\delta H} f(X,Y,Z,t)\,\mathrm{d}S = \iint_{\delta H} f(x',y',z',t)\,\mathrm{d}s = \oint_{\Gamma}\mathrm{d}l\int_{0}^{\eta-v} f(x',y',z'+Z_0,t)\,\mathrm{d}z'/\cos(\alpha+\gamma) \tag{12}$$

in which α is the inclining angle of hull at the waterline and γ is the angle due to the rotations of the ship, as illustrated on the right of Figure 2. The angles α and γ are defined by

$$\alpha = \tan^{-1}(-n_3/\sqrt{1-n_3^2}) \quad \text{and} \quad \gamma = -\mathbf{e}_3 \cdot (\boldsymbol{\theta} \wedge \boldsymbol{n}) = \theta_2 n_1 - \theta_1 n_2 \tag{13}$$

in relation with the normal vector $\boldsymbol{n} = (n_1, n_2, n_3)$ on the ship hull at the waterline. The vertical displacement v of the waterline Γ is positive if it is over the mean free surface $z' = -Z_0$ ($v < 0$ in Figure 2) like the wave elevation η. The integral on the intermittent surface is transformed to the integral $\mathrm{d}l$ along the waterline and the vertical integral $\mathrm{d}z'$ between the mean waterline $z' = -Z_0$ and $z = \eta - v - Z_0$ which is translated to $z' + Z_0 = 0$ to $z' + Z_0 = \eta - v$. The factor $1/\cos(\alpha + \gamma)$ is introduced to correct the inclined distance between η and v by assuming the hull inclination with respect to the vertical line is straight in the vicinity of waterline. It is important to note that the hull surface H in (11) and the waterline Γ in (12) are at their instantaneous position. Both integrals are to be evaluated at their mean position so that the integrand function $f(x', y', z', t)$ is to be expressed at the mean position by using the Taylor expansions. If the normal vector is involved, the variations due to ship rotations are to be taken into account.

II – 4 Decompositions according to the pertubation analysis

By assuming a small parameter ϵ to be proportional to wave steepness, the time-harmonic unsteady motions $(\boldsymbol{\xi}, \boldsymbol{\theta})$ are expressed by the sum of

$$(\boldsymbol{\xi}, \boldsymbol{\theta}) = \underbrace{\cancelto{0}{(\boldsymbol{\xi}^s, \boldsymbol{\theta}^s)}}_{O(1)} + \Re\text{e}\big\{\underbrace{\xi_0(\boldsymbol{\xi}_1, \boldsymbol{\theta}_1)}_{O(\epsilon)}\,\mathrm{e}^{-\mathrm{i}\omega t}\big\} + \underbrace{\xi_0^2(\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\theta}})}_{O(\epsilon^2)} + \Re\text{e}\big\{\underbrace{\xi_0^2(\boldsymbol{\xi}_2, \boldsymbol{\theta}_2)}_{O(\epsilon^2)}\,\mathrm{e}^{-\mathrm{i}2\omega t}\big\} + O(\epsilon^3) \tag{14}$$

in which $(\boldsymbol{\xi}^s, \boldsymbol{\theta}^s)$ represent the mean displacements due to the steady flow.

The position of ship (in calm water) is modified due to the steady forces induced by the steady flow around the ship. The horizontal steady displacements (in surge/sway/yaw) should be corrected by the propulsion action combined with rudders. The vertical mean displacements like sinkage and trim are possible to balance the force in heave and moment in pitch/roll by hydrostatic stiffness. In any case, we can consider the reference systems associated with the equilibrium position of ship in steady flows so that $(\boldsymbol{\xi}^s, \boldsymbol{\theta}^s)$ are imposed to be zero without loss of generality.

In unsteady flow, ξ_0 denotes the incoming wave amplitude. The first term on the right hand side of (14) denotes the linear component oscillatory at the encounter frequency ω. The second term is the mean displacement and third is of oscillatory at the double encounter frequency. Both the second and third terms are of second order. The terms of third order or higher are not explicitly denoted in (14). It worth noting that there is a component of mean displacements (sinkage and trim, for example) due to the steady flow which are not accounted here. Nor the second-order motions $(\boldsymbol{\xi}_2, \boldsymbol{\theta}_2)$ at the double frequency since we are interested here the second-order mean values, so that we shall write the first-motion amplitudes $(\boldsymbol{\xi}_1, \boldsymbol{\theta}_1)$ by $(\boldsymbol{\xi}, \boldsymbol{\theta})$ for the sake of simplicity [1].

The same notations as in (14) are used for any other quantities of wave fields like wave elevations and velocity potentials, and physical quantities such as pressures and forces, etc. In particular, the total velocity potential Φ scaled by $\sqrt{gL^3}$ and its gradient $\nabla\Phi$ scaled by $\sqrt{gL}$ are written by

$$\begin{aligned} \Phi &= F_\mathrm{r}\phi^s + \Re\text{e}\big\{(\xi_0/L)\phi\,\mathrm{e}^{-\mathrm{i}\omega t}\big\} + (\xi_0/L)^2\bar{\psi} + \cdots \\ \nabla\Phi &= F_\mathrm{r}\mathbf{w} + \Re\text{e}\big\{(\xi_0/L)\nabla\phi\,\mathrm{e}^{-\mathrm{i}\omega t}\big\} + (\xi_0/L)^2\nabla\bar{\psi} + \cdots \end{aligned} \tag{15}$$

where $F_\mathrm{r} = U/\sqrt{gL}$ is the Froude number to represent the forward U scaled with the gravity acceleration g and ship length L. The velocity potential of steady flow is denoted by ϕ^s and

1. without confusion with the second-order mean displacements $(\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\theta}})$.

its gradient by $\mathbf{w} = \nabla\phi^s$ in their non-dimensional form. The non-dimensional potential of linear unsteady flow is denoted in ϕ in (15). The second-order potential $\bar{\psi}$ represents the steady component which could contribute to the second-order steady loads as shown in [4]. There should have a component of second order oscillatory at the double frequency which is ignored here.

Finally, the product of two linear time-harmonic quantities $(A, B) = \Re\{(a, b)\mathrm{e}^{-\mathrm{i}\omega t}\}$ is usually written by

$$AB = \Re\{a\mathrm{e}^{-\mathrm{i}\omega t}\}\Re\{b\mathrm{e}^{-\mathrm{i}\omega t}\} = \overline{AB} + \frac{1}{2}\Re\{(ab)\mathrm{e}^{-2\mathrm{i}\omega t}\} \quad \text{with} \quad \overline{AB} = \frac{1}{4}(ab^* + a^*b) \tag{16}$$

with (a^*, b^*) being the complex conjugate of (a, b), respectively. Again, since we are interested here only on the mean part of the second-order mean value, the product $\overline{AB}$ is written as ab for the sake of simplicity.

III – Nearfield formulations

The direct integration of fluid pressure on the instantaneous hull surface gives wave loading on ships. Since the pressure obtained by applying Bernoulli's equation involves only wave fields in the vicinity of ship hull, the formulation is called the nearfield formulation. We write the fluid pressure P scaled by $(\rho g L)$, the wave elevation η above the waterline and its vertical displacement v scaled by L, the vector $\boldsymbol{X}$ of displacement at any point on the ship hull scaled by L, and the forces $\boldsymbol{F}$ scaled by $\rho g L^3$, in the translating coordinate system as

$$\begin{cases} P = P_0 + \Re\{(\xi_0/L)P_1\mathrm{e}^{-\mathrm{i}\omega t}\} + (\xi_0/L)^2P_2 + \cdots \\ \eta = F_\mathrm{r}^2\eta_0 + \Re\{(\xi_0/L)\eta_1\mathrm{e}^{-\mathrm{i}\omega t}\} + (\xi_0/L)^2\eta_2 + \cdots \\ v = \Re\{(\xi_0/L)v_1\mathrm{e}^{-\mathrm{i}\omega t}\} + (\xi_0/L)^2v_2 + \cdots \\ \boldsymbol{X} = \Re\{(\xi_0/L)\boldsymbol{X}_1\mathrm{e}^{-\mathrm{i}\omega t}\} + (\xi_0/L)^2\boldsymbol{X}_2 + \cdots \\ \boldsymbol{F} = \boldsymbol{F}_0 + \Re\{(\xi_0/L)\boldsymbol{F}_1\mathrm{e}^{-\mathrm{i}\omega t}\} + (\xi_0/L)^2\boldsymbol{F}_2 + \cdots \end{cases} \tag{17}$$

with the zeroth-order steady components $(P_0, \eta_0, \boldsymbol{F}_0)$, the linear component $(P_1, \eta_1, v_1, \boldsymbol{X}_1, \boldsymbol{F}_1)$, oscillating at the encounter frequency, and the second-order mean component $(P_2, \eta_2, v_2, \boldsymbol{X}_2, \boldsymbol{F}_2)$. The pressure components (P_0, P_1, P_2) and (η_0, η_1, η_2) evaluated at $z = -Z_0$ are given by

$$\begin{cases} P_0 = -(z + Z_0) + F_\mathrm{r}^2P_0^s \\ P_1 = \mathrm{i}\omega\phi - F_\mathrm{r}\mathbf{w}\cdot\nabla\phi \\ P_2 = -\dfrac{1}{2}\nabla\phi\cdot\nabla\phi - F_\mathrm{r}\mathbf{w}\cdot\nabla\bar{\psi} \end{cases} \qquad \begin{cases} \eta_0 = P_0|_{z=-Z_0} = -\dfrac{1}{2}(\mathbf{w}\cdot\mathbf{w} - 1) \\ \eta_1 = P_1|_{z=-Z_0} = \mathrm{i}\omega\phi - F_\mathrm{r}\mathbf{w}\cdot\nabla\phi \\ \eta_2 = P_2|_{z=-Z_0} = -\dfrac{1}{2}\nabla\phi\cdot\nabla\phi - F_\mathrm{r}\mathbf{w}\cdot\nabla\bar{\psi} \end{cases} \tag{18}$$

respectively. The zeroth-order pressure P_0 includes the pressure $P_0^s = -\frac{1}{2}(\mathbf{w}\cdot\mathbf{w} - 1)$ associated with the steady flow in addition to the hydrostatic part. The displacement vectors $(\boldsymbol{X}_1, \boldsymbol{X}_2)$ and the vertical displacements (v_1, v_2) evaluated at $z = -Z_0$ are obtained by

$$\begin{cases} \boldsymbol{X}_1 = \boldsymbol{\xi} + \mathbb{T}_1(\boldsymbol{\theta})\boldsymbol{r} \\ \boldsymbol{X}_2 = \bar{\boldsymbol{\xi}} + [\mathbb{T}_1(\bar{\boldsymbol{\theta}}) + \mathbb{T}_2(\boldsymbol{\theta})]\boldsymbol{r} \end{cases} \qquad \begin{cases} v_1 = \xi_3 + \theta_1 y - \theta_2 x \\ v_2 = \bar{\xi}_3 + \bar{\theta}_1 y - \bar{\theta}_2 x + [\theta_3\theta_1 x + \theta_2\theta_3 y + (\theta_1^2 + \theta_2^2)Z_0]/2 \end{cases} \tag{19}$$

by using the first-order components $(\boldsymbol{\xi}, \boldsymbol{\theta})$ which is understood to be $(\boldsymbol{\xi}_1, \boldsymbol{\theta}_1)$ in (14), and the second-order mean component $(\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\theta}})$ of ship motions. The transfer matrices $(\mathbb{T}_1, \mathbb{T}_2)$ are defined in (6) and (7), respectively.

Now we compute the loads $\boldsymbol{F}$ by the integration of fluid pressure on the ship hull at its instantaneous position $\mathcal{H}$ which is composed of the hull H below the mean waterline and the intermittent surface δH around the waterline Γ. The integral on the instantaneous hull H is further transformed to the integral on $H_0(\boldsymbol{X}_0)$ at its mean position, i.e., the mean position under the steady flow. In the same way, the integral on the instantaneous intermittent surface δH is transformed in the integral on the mean intermittent surface : along the mean waterline Γ_0 and vertical line between the mean waterline and the wave crest $\eta - v$ over the waterline which is vertically displaced v according to (12) :

$$
\begin{aligned}
\boldsymbol{F} &= -\iint_{\mathcal{H}} P\boldsymbol{n}\,\mathrm{d}s = -\iint_{H} P\boldsymbol{n}\,\mathrm{d}s - \iint_{\delta H} P\boldsymbol{n}\,\mathrm{d}s \\
&= -\iint_{H_0} \Big\{\big[1 + (\boldsymbol{X}\cdot\nabla) + \frac{1}{2}(\boldsymbol{X}\cdot\nabla)^2\big]P\Big\}\Big\{\big[\mathbb{I} + \mathbb{T}_1 + \bar{\mathbb{T}}_1 + \mathbb{T}_2\big]\boldsymbol{n}\Big\}\,\mathrm{d}s \\
&\quad - \oint_{\Gamma_0} \mathrm{d}l \int_0^{\eta - v} \Big\{(1 + z\partial_z)\big[1 + (\boldsymbol{X}\cdot\nabla) + \frac{1}{2}(\boldsymbol{X}\cdot\nabla)^2\big]P\Big\}\Big\{\big[\mathbb{I} + \mathbb{T}_1 + \bar{\mathbb{T}}_1 + \mathbb{T}_2\big]\boldsymbol{n}\Big\}\frac{\mathrm{d}z}{\cos(\alpha+\gamma)}
\end{aligned} \tag{20}
$$

in which the Taylor expansions of $P(H)$ by $P(H_0)$ on H_0 and of $P(\Gamma)$ by $P(\Gamma_0)$ along Γ_0 are used. In (20), the force components are expressed in the translation system with the reference point at the origin o'. Since H_0 and Γ_0 are at their mean position ($x' = x, y' = y$) at which the ship-fixed coordinate system can be used with a slight modification of $z = z' + Z_0$. Furthermore, the matrix $\mathbb{T}_1 = \mathbb{T}_1(\boldsymbol{\theta})$ is associated with the first-order rotation $\boldsymbol{\theta}$ while $\bar{\mathbb{T}}_1 = \mathbb{T}_1(\bar{\boldsymbol{\theta}})$ is associated with the second-order rotation $\bar{\theta}$ and $\mathbb{T}_2 = \mathbb{T}_2(\boldsymbol{\theta})$ with the product of first-order rotations. Both $\bar{\mathbb{T}}_1$ and $\mathbb{T}_2$ are of second order. The variations of the normal vector $\boldsymbol{n}$ (oriented positively into the fluid) due to the rotations are taken into account as well. By identifying the components defined in (17) and those in (20), we obtain

$$
\begin{aligned}
\boldsymbol{F}_0 &= -\iint_{H_0} P_0\boldsymbol{n}\,\mathrm{d}s - F_{\mathrm{r}}^2 \oint_{\Gamma_0} \Big\{\eta_0 P_0 + F_{\mathrm{r}}^2 \frac{1}{2}\eta_0^2 \partial_z P_0\Big\}\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l \\
\boldsymbol{F}_1 &= -\iint_{H_0} (P_1 + \boldsymbol{X}_1\cdot\nabla P_0)\boldsymbol{n}\,\mathrm{d}s - \mathbb{T}_1 \iint_{H_0} P_0\boldsymbol{n}\,\mathrm{d}s \\
&\quad - \oint_{\Gamma_0} \Big\{(\eta_1 - v_1)P_0 + F_{\mathrm{r}}^2\eta_0(P_1 + \boldsymbol{X}_1\cdot\nabla P_0 + P_0\mathbb{T}_1) + (\eta_1 - v_1)F_{\mathrm{r}}^2\eta_0\partial_z P_0 + \frac{1}{2}F_{\mathrm{r}}^4\eta_0^2\partial_z P_1 \\
&\qquad\qquad + F_{\mathrm{r}}^2\gamma\tan\alpha\big[\eta_0 P_0 + F_{\mathrm{r}}^2\frac{1}{2}\eta_0^2\partial_z P_0\big]\Big\}\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned} \tag{21}
$$

for the zeroth-order and first-order forces. The second-order forces are expressed by

$$
\begin{aligned}
\boldsymbol{F}_2 &= -\iint_{H_0} \big[P_2 + \boldsymbol{X}_1\cdot\nabla P_1 + \frac{1}{2}(\boldsymbol{X}_1\cdot\nabla)^2 P_0 + \boldsymbol{X}_2\cdot\nabla P_0\big]\boldsymbol{n}\,\mathrm{d}s \\
&\qquad - \mathbb{T}_1 \iint_{H_0} (P_1 + \boldsymbol{X}_1\cdot\nabla P_0)\boldsymbol{n}\,\mathrm{d}s - (\bar{\mathbb{T}}_1 + \mathbb{T}_2)\iint_{H_0} P_0\boldsymbol{n}\,\mathrm{d}s \\
&\quad - \oint_{\Gamma_0} \Big\{(\eta_2 - v_2)P_0 + (\eta_1 - v_1)(P_1 + \boldsymbol{X}_1\cdot\nabla P_0 + P_0\mathbb{T}_1 + F_{\mathrm{r}}^2\eta_0\partial_z P_1) \\
&+ F_{\mathrm{r}}^2\eta_0\big[P_2 + \boldsymbol{X}_1\cdot\nabla P_1 + \frac{1}{2}(\boldsymbol{X}_1\cdot\nabla)^2 P_0 + \boldsymbol{X}_2\cdot\nabla P_0 + F_{\mathrm{r}}^2(\eta_1 - v_1)(\boldsymbol{X}_1\cdot\nabla)\partial_z P_0\big] \\
&\quad + \big[\frac{1}{2}(\eta_1 - v_1)^2 + F_{\mathrm{r}}^2\eta_0(\eta_2 - v_2)\big]\partial_z P_0 + \gamma^2(\frac{1}{2} + \tan^2\alpha)F_{\mathrm{r}}^2\eta_0\big[P_0 + \frac{1}{2}F_{\mathrm{r}}^2\eta_0\partial_z P_0\big] \\
&\quad + \gamma\tan\alpha\big[(\eta_1 - v_1)P_0 + F_{\mathrm{r}}^2\eta_0(P_1 + \boldsymbol{X}_1\cdot\nabla P_0 + P_0\mathbb{T}_1) + (\eta_1 - v_1)F_{\mathrm{r}}^2\eta_0\partial_z P_0\big]\Big\}\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned} \tag{22}
$$

with γ given in (13). The terms associated with γ are derived by using the development

$$
\frac{1}{\cos(\alpha+\gamma)} = \frac{1}{\cos\alpha}\Big\{1 + \gamma\tan\alpha + \gamma^2(\frac{1}{2} + \tan^2\alpha)\Big\} + O(\gamma^3) \tag{23}
$$

The steady pressure $P_0 = P_0^h + P_0^s$ defined in (18) includes the hydrostatic part $P_0^h = -(z + Z_0)$ and that $P_0^s = -\frac{1}{2}(\mathbf{w}\cdot\mathbf{w} - 1)$ due to the steady flow $\mathbf{w}$. Introducing the composition of P_0 in (22) and using the identities

$$
P_0^s = \eta_0 \;; \quad P_1 = \eta_1 \quad \text{and} \quad P_2 = \eta_2 \tag{24}
$$

on the mean free surface $z=-Z_0$, we have

$$
\begin{aligned}
\boldsymbol{F}_2 = &-\iint_{H_0}\Big\{P_2+\boldsymbol{X}_1\cdot\nabla P_1+\big[P_1-(\boldsymbol{X}_1\cdot\mathbf{e}_3)\big]\mathbb{T}_1\Big\}\boldsymbol{n}\,\mathrm{d}s-\frac{1}{2}\oint_{\Gamma_0}(\eta_1-v_1)^2\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l\\
&+\iint_{H_0}\big[\boldsymbol{X}_2\cdot\mathbf{e}_3+(z+Z_0)(\bar{\mathbb{T}}_1+\mathbb{T}_2)\big]\boldsymbol{n}\,\mathrm{d}s\\
&-F_\mathrm{r}^2\iint_{H_0}\big[(\boldsymbol{X}_1\cdot\nabla P_0^s)\mathbb{T}_1+\frac{1}{2}(\boldsymbol{X}_1\cdot\nabla)^2P_0^s+\boldsymbol{X}_2\cdot\nabla P_0^s+P_0^s(\bar{\mathbb{T}}_1+\mathbb{T}_2)\big]\boldsymbol{n}\,\mathrm{d}s\\
&-F_\mathrm{r}^2\oint_{\Gamma_0}\Big\{(\eta_1-v_1)\boldsymbol{X}_1\cdot\nabla P_0^s+\eta_0\big[(\eta_2-v_2)+(\eta_1-v_1)(\mathbb{T}_1+\partial_zP_1)\big]\\
&+\eta_0\big[\boldsymbol{X}_1\cdot\nabla P_1+\frac{1}{2}F_\mathrm{r}^2(\boldsymbol{X}_1\cdot\nabla)^2P_0^s+F_\mathrm{r}^2\boldsymbol{X}_2\cdot\nabla P_0^s+F_\mathrm{r}^2(\eta_1-v_1)(\boldsymbol{X}_1\cdot\nabla)\partial_zP_0^s\big]\\
&+\big[\frac{1}{2}(\eta_1-v_1)^2+F_\mathrm{r}^2\eta_0(\eta_2-v_2)\big]\partial_zP_0^s+\gamma^2(\frac{1}{2}+\tan^2\alpha)F_\mathrm{r}^2\frac{1}{2}\eta_0^2(F_\mathrm{r}^2\partial_zP_0^s+1)\\
&+(\gamma\tan\alpha)\eta_0\big[2(\eta_1-v_1)+\boldsymbol{X}_1\cdot\nabla P_0^s+\mathbb{T}_1+(\eta_1-v_1)(F_\mathrm{r}^2\partial_zP_0^s-1)\big]\Big\}\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned}
\tag{25}
$$

in which (P_1,P_2) are defined in (18) including explicitly the terms associated with the forward speed (Froude number). The first line on the right hand side of (25) is classical as it looks like the nearfield formulation of drift forces at zero speed as presented in [10], [6] and [8]. The second line was missing in [10] but well documented in [6] and [8] in the case of zero speed. As shown in [6] and [8], the hull integral of all terms in the second line does not contribute to the horizontal components of second-order forces. Other terms from the third line are associated with the steady flow $\mathbf{w}=\nabla\phi^s$ on which $(\eta_0,\nabla P_0^s)$ are dependent. In summary, the horizontal components of second-order forces can be written by

$$
\boldsymbol{F}_N=\boldsymbol{F}_2=\boldsymbol{F}_N^C+F_\mathrm{r}^2(\boldsymbol{F}_N^H+\boldsymbol{F}_N^W)
\tag{26}
$$

with the classical component

$$
\begin{aligned}
\boldsymbol{F}_N^C=&\iint_{H_0}\Big\{(\frac{1}{2}\nabla\phi\cdot\nabla\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\bar{\psi})-\boldsymbol{X}_1\cdot\nabla(\mathrm{i}\omega\phi-F_\mathrm{r}\mathbf{w}\cdot\nabla\phi)\\
&-\big[(\mathrm{i}\omega\phi-F_\mathrm{r}\mathbf{w}\cdot\nabla\phi)-(\boldsymbol{X}_1\cdot\mathbf{e}_3)\big]\mathbb{T}_1\Big\}\boldsymbol{n}\,\mathrm{d}s-\frac{1}{2}\oint_{\Gamma_0}(\eta_1-v_1)^2\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned}
\tag{27}
$$

which can also be written by

$$
\begin{aligned}
\boldsymbol{F}_N^C=&\iint_{H_0}\Big\{(\frac{1}{2}\nabla\phi\cdot\nabla\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\bar{\psi})+\boldsymbol{X}_1\cdot\nabla(-\mathrm{i}\omega\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\phi)\\
&+(-\mathrm{i}\omega\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\phi)\mathbb{T}_1\Big\}\boldsymbol{n}\,\mathrm{d}s+\frac{1}{2}\oint_{\Gamma_0}\eta_1(2v_1-\eta_1)\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned}
\tag{28}
$$

for the horizontal components in [8] and used in [1] to obtain another equivalent formulation. The equivalent nearfield formulation is written in a more compact form

$$
\boldsymbol{F}_N^C=\iint_{H_0}\Big\{(\frac{1}{2}\nabla\phi\cdot\nabla\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\bar{\psi})\boldsymbol{n}+(\boldsymbol{X}_1\cdot\boldsymbol{n})\nabla(-\mathrm{i}\omega\phi+F_\mathrm{r}\mathbf{w}\cdot\nabla\phi)\Big\}\,\mathrm{d}s-\frac{1}{2}\oint_{\Gamma_0}\eta_1^2\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\tag{29}
$$

by using the identity (eq.57) in [1]. The component $\boldsymbol{F}_2^H$ associated with the steady flow P_0^s is written by the component of integral on the hull

$$
\boldsymbol{F}_N^H=-\iint_{H_0}\Big\{(\boldsymbol{X}_1\cdot\nabla P_0^s)\mathbb{T}_1+\frac{1}{2}(\boldsymbol{X}_1\cdot\nabla)^2P_0^s+\boldsymbol{X}_2\cdot\nabla P_0^s+P_0^s(\bar{\mathbb{T}}_1+\mathbb{T}_2)\Big\}\boldsymbol{n}\,\mathrm{d}s
\tag{30}
$$

and the component $\boldsymbol{F}_2^W$ of integral along the waterline by

$$
\begin{aligned}
\boldsymbol{F}_N^W = &-\oint_{\Gamma_0}\Big\{(\eta_1-v_1)\boldsymbol{X}_1\cdot\nabla P_0^s+\eta_0\big[2(\eta_2-v_2)+(\eta_1-v_1)(\mathbb{T}_1+\partial_z P_1)\big]\\
&+\eta_0\big[\boldsymbol{X}_1\cdot\nabla P_1+\frac{1}{2}F_\mathrm{r}^2(\boldsymbol{X}_1\cdot\nabla)^2P_0^s+F_\mathrm{r}^2\boldsymbol{X}_2\cdot\nabla P_0^s+F_\mathrm{r}^2(\eta_1-v_1)(\boldsymbol{X}_1\cdot\nabla)\partial_z P_0^s\big]\\
&+\big[\frac{1}{2}(\eta_1-v_1)^2+F_\mathrm{r}^2\eta_0(\eta_2-v_2)\big]\partial_z P_0^s+\gamma^2(\frac{1}{2}+\tan^2\alpha)F_\mathrm{r}^2\frac{1}{2}\eta_0^2(F_\mathrm{r}^2\partial_z P_0^s+1)\\
&+(\gamma\tan\alpha)\eta_0\big[2(\eta_1-v_1)+\boldsymbol{X}_1\cdot\nabla P_0^s+\mathbb{T}_1+(\eta_1-v_1)(F_\mathrm{r}^2\partial_z P_0^s-1)\big]\Big\}\frac{\boldsymbol{n}}{\cos\alpha}\,\mathrm{d}l
\end{aligned}
\tag{31}
$$

Both $\boldsymbol{F}_N^H$ and $\boldsymbol{F}_N^W$ are associated with the steady flow P_0^s and contribute to the total mean forces with the factor F_r^2 in (26).

IV – Midfield formulations

We consider a finite domain D limited by the instantaneous ship hull $\mathcal{H}$, the control surface $\mathcal{C}$ and the part of free surface $\mathcal{F}$ in between. In this domain D, the total fluid flow is represented by the velocity $\boldsymbol{V}=\nabla\Phi$ with the total velocity potential Φ and $\nabla\Phi$ defined in (15). The linear momentum $\boldsymbol{M}$ contained in the domain scaled by $\rho\sqrt{gL}$ is written by

$$
\boldsymbol{M}=\iiint_D \boldsymbol{V}\,\mathrm{d}v=\iiint_D \nabla\Phi\,\mathrm{d}v \tag{32}
$$

and the rate of change of the linear momentum

$$
\frac{d}{dt}\boldsymbol{M}=\iiint_D\frac{\partial}{\partial t}\boldsymbol{V}\,\mathrm{d}v+\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\boldsymbol{V}U_n\,\mathrm{d}s \tag{33}
$$

according to Reynolds transport theorem. Introducing $\boldsymbol{V}=\nabla\Phi$ in (33) and using the divergence theorem to the volume integral, the rate of momentum change can be written as

$$
\frac{d}{dt}\boldsymbol{M}=\iiint_D\nabla\Phi_t\,\mathrm{d}v+\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\nabla\Phi U_n\,\mathrm{d}s=\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\big[\Phi_t\boldsymbol{n}+\nabla\Phi U_n\big]\,\mathrm{d}s \tag{34}
$$

in which we have

$$
\Phi_t=-P-(z+Z_0)-\frac{1}{2}\nabla\Phi\cdot\nabla\Phi+\frac{1}{2}F_\mathrm{r}^2 \tag{35}
$$

according to Bernoulli's equation, so that

$$
\begin{aligned}
\frac{d}{dt}\boldsymbol{M}&=\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\Big\{\big[-P-(z+Z_0)-\frac{1}{2}\nabla\Phi\cdot\nabla\Phi+\frac{1}{2}F_\mathrm{r}^2\big]\boldsymbol{n}+\nabla\Phi U_n\Big\}\,\mathrm{d}s\\
&=\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\Big\{\big[-P-(z+Z_0)\big]\boldsymbol{n}-\nabla\Phi(\Phi_n-U_n)\Big\}\,\mathrm{d}s
\end{aligned}
\tag{36}
$$

by using

$$
\frac{1}{2}F_\mathrm{r}^2\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\boldsymbol{n}\,\mathrm{d}s=0 \tag{37}
$$

and

$$
\frac{1}{2}\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}(\nabla\Phi\cdot\nabla\Phi)\boldsymbol{n}\,\mathrm{d}s=\iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}}\Phi_n\nabla\Phi\,\mathrm{d}s \tag{38}
$$

which is shown by (eq.65) in [1]. The velocity U_n in (34) and (36) is the velocity component in the normal direction of the boundary surfaces (ship hull $\mathcal{H}$, the free surface $\mathcal{F}$ and the control surface $\mathcal{C}$). The fact that $P = 0$ on $\mathcal{F}$, $U_n = \Phi_n$ on $\mathcal{H} \cup \mathcal{F}$ by the kinematic condition, and $U_n = 0$ on $\mathcal{C}$ which is imposed to be fixed, yields

$$\begin{aligned} \boldsymbol{F} = -\iint_{\mathcal{H}} P\boldsymbol{n}\,\mathrm{d}s &= \frac{d}{dt}\boldsymbol{M} + \iint_{\mathcal{H}\cup\mathcal{F}\cup\mathcal{C}} (z+Z_0)\boldsymbol{n}\,\mathrm{d}s + \iint_{\mathcal{C}} \big[P\boldsymbol{n} + \nabla\Phi(\Phi_n - U_n)\big]\,\mathrm{d}s \\ &= \frac{d}{dt}\boldsymbol{M} + \mathrm{V}\mathbf{e}_3 + \iint_{\mathcal{C}} \big[P\boldsymbol{n} + \Phi_n\nabla\Phi\big]\,\mathrm{d}s \end{aligned} \tag{39}$$

with V the fluid volume contained in the domain. The term $\mathrm{V}\mathbf{e}_3$ is the buoyant force balanced by the gravity force. The horizontal components of mean forces are

$$< F_x, F_y >= \left\langle \iint_{\mathcal{C}} \big[P(n_1,n_2) + \Phi_n(\Phi_x,\Phi_y)\big]\,\mathrm{d}s \right\rangle \tag{40}$$

since

$$\left\langle \frac{d}{dt}\boldsymbol{M} \right\rangle = 0 \tag{41}$$

according to the conservation principle of the linear momentum for periodic motions. The mean drift forces are then expressed by the integral over the instantaneous control surface $\mathcal{C} = C + \delta C$ with C the part below the mean free surface $z \le -Z_0$ and the part from the mean free surface to the instantaneous free surface represented by the wave elevation η. The integral on the right hand side of (40) can then be written as

$$< F_x, F_y >= \left\langle \iint_{C} \big[P(n_1,n_2) + \Phi_n(\Phi_x,\Phi_y)\big]\mathrm{d}s \right\rangle + \left\langle \oint_{\Gamma_c} \mathrm{d}l \int_0^{\eta} \big[P(n_1,n_2) + \Phi_n(\Phi_x,\Phi_y)\big]\mathrm{d}z \right\rangle \tag{42}$$

Using the decompositions (17) for P and (15) for Φ, in (42), the second-order mean forces with the notation $\boldsymbol{F}_M =< F_x, F_y >$ can be distilled and reported by

$$\begin{aligned} \boldsymbol{F}_M = &\iint_C \big[(-\frac{1}{2}\nabla\phi\cdot\nabla\phi - F_{\mathrm{r}}\mathbf{w}\cdot\nabla\bar{\psi})(n_1,n_2) + \phi_n(\phi_x,\phi_y) + F_{\mathrm{r}}\bar{\psi}_n(\phi_x^s,\phi_y^s)\big]\mathrm{d}s \\ &+ \frac{1}{2}\oint_{\Gamma_c} \eta_1^2(n_1,n_2)\,\mathrm{d}l + F_{\mathrm{r}}\oint_{\Gamma_c} \eta_1\big[\phi_n(\phi_x^s,\phi_y^s) + (\phi_x,\phi_y)(\mathbf{w}\cdot\boldsymbol{n})\big]\,\mathrm{d}l \\ &+ F_{\mathrm{r}}^2 \oint_{\Gamma_c} \Big\{\eta_0\big[2\eta_2(n_1,n_2) + \phi_n(\phi_x,\phi_y) + F_{\mathrm{r}}\bar{\psi}_n(\phi_x^s,\phi_y^s)\big] + \eta_2(\mathbf{w}\cdot\boldsymbol{n})(\phi_x^s,\phi_y^s)\Big\}\,\mathrm{d}l \end{aligned} \tag{43}$$

which could again written as

$$\boldsymbol{F}_M = \boldsymbol{F}_M^C + F_{\mathrm{r}}^2\boldsymbol{F}_M^W \tag{44}$$

with the component $\boldsymbol{F}_M^C$ defined by

$$\begin{aligned} \boldsymbol{F}_M^C = &\iint_C \big[(-\frac{1}{2}\nabla\phi\cdot\nabla\phi - F_{\mathrm{r}}\mathbf{w}\cdot\nabla\bar{\psi})(n_1,n_2) + \phi_n(\phi_x,\phi_y) + F_{\mathrm{r}}\bar{\psi}_n(\phi_x^s,\phi_y^s)\big]\mathrm{d}s \\ &+ \frac{1}{2}\oint_{\Gamma_c} \eta_1^2(n_1,n_2)\,\mathrm{d}l + F_{\mathrm{r}}\oint_{\Gamma_c} \eta_1\big[\phi_n(\phi_x^s,\phi_y^s) + (\phi_x,\phi_y)(\mathbf{w}\cdot\boldsymbol{n})\big]\,\mathrm{d}l \end{aligned} \tag{45}$$

and the component associated with the elevation η_0 of steady flow

$$\boldsymbol{F}_M^W = \oint_{\Gamma_c} \Big\{\eta_0\big[2\eta_2(n_1,n_2) + \phi_n(\phi_x,\phi_y) + F_{\mathrm{r}}\bar{\psi}_n(\phi_x^s,\phi_y^s)\big] + \eta_2(\mathbf{w}\cdot\boldsymbol{n})(\phi_x^s,\phi_y^s)\Big\}\,\mathrm{d}l \tag{46}$$

The midfield formulation (43) is applicable to any control surface $\mathcal{C}$ surrounding the ship. The control surface $\mathcal{C}$ can be put back to the hull $\mathcal{H}$ as suggested in [8] and care about the sign of normal vector. Eventually, the control surface $\mathcal{C}$ can be put to the infinity.

V – Farfield formulations

We consider a vertical cylindrical surface of radius $R \gg 1$ surrounding the ship hull $\mathcal{H}$. On this cylindrical surface, it can be shown that $\boldsymbol{F}_M^W$ associated with η_0 is nil. Furthermore, the second-order potential $\bar{\psi}$ is assumed to be zero [2] as in [5] and [7]. By using $\phi^s = -x$ and $\mathbf{w} \cdot \boldsymbol{n} = -n_1 = \cos\gamma$ with γ the polar angle. The farfield formulation can be derived from (45) and written by

$$\begin{aligned} < F_x, F_y > = \frac{1}{4}\int_{-\infty}^{0} \mathrm{d}z \int_{-\pi}^{\pi} R\big[(\nabla\phi\cdot\nabla\phi^*)(n_1,n_2) - (\phi_x,\phi_y)\phi_n^* - (\phi_x^*,\phi_y^*)\phi_n)\big]\,\mathrm{d}\gamma \\ -\frac{1}{4}\int_{-\pi}^{\pi} R(\omega^2\phi\phi^* - F_\mathrm{r}^2\phi_x\phi_x^*)(n_1,n_2)\,\mathrm{d}\gamma \end{aligned} \tag{47}$$

in which $(\varphi^*, \nabla\varphi^*, \varphi_n^*)$ are the complex conjugate of $(\varphi, \nabla\varphi, \varphi_n)$, respectively. The first-order potential $\phi = \phi^P + \phi_0$ is composed of the part of incoming waves ϕ_0 and the perturbation part ϕ^P which is expressed asymptotically. By using the stationary phase method, the first farfield formulation for $< F_x >$ was derived by Maruo [7]. This pioneer work has been extended in [5] by applying Parseval's theorem in the Fourier-transform theory, to the transverse load $< F_y >$ and yaw moment $< M_z >$. Here the farfield formulation $< F_x, F_y >$ given in [5] is reported by

$$\begin{aligned} <F_x> &= \frac{4\pi}{F_\mathrm{r}^2}\Big(-\int_{-\infty}^{\alpha_1} + \int_{\alpha_2}^{\alpha_3} + \int_{\alpha_4}^{\infty}\Big)\big[|C(\alpha)|^2 + |S(\alpha)|^2\big]\frac{(\alpha - F_\mathrm{r}^2 k_0\cos\beta)}{\sqrt{k^2-\alpha^2}}k\,\mathrm{d}\alpha \\ <F_y> &= \frac{4\pi}{F_\mathrm{r}^2}\Big(-\int_{-\infty}^{\alpha_1} + \int_{\alpha_2}^{\alpha_3} + \int_{\alpha_4}^{\infty}\Big)\Big\{\big[|C(\alpha)|^2 + |S(\alpha)|^2\big]\frac{F_\mathrm{r}^2 k_0\sin\beta}{\sqrt{k^2-\alpha^2}} - \Im\mathrm{m}\{2C(\alpha)S(\alpha)\}\Big\}k\,\mathrm{d}\alpha \end{aligned} \tag{48}$$

with $k = (\tau + \alpha)^2$ the wavenumber of unsteady ship waves and (k_0, β) the (wavenumber,heading) of incoming waves. The complex Kochin functions $C(\alpha)$ and $S(\alpha)$ are defined in [5] by

$$\begin{aligned} C(\alpha) &= \frac{1}{4\pi}\iint_{H_0}(\phi_n^P - \phi^P\partial_n)\mathrm{e}^{kz-\mathrm{i}x\alpha}\cos(y\sqrt{k^2-\alpha^2})\,\mathrm{d}s \\ S(\alpha) &= \frac{1}{4\pi}\iint_{H_0}(\phi_n^P - \phi^P\partial_n)\mathrm{e}^{kz-\mathrm{i}x\alpha}\sin(y\sqrt{k^2-\alpha^2})\,\mathrm{d}s \end{aligned} \tag{49}$$

and the values of $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ by

$$\begin{aligned} \alpha_1 &= -(1+2\tau+\sqrt{1+4\tau})/2 \qquad & \alpha_2 &= -(1+2\tau-\sqrt{1+4\tau})/2 \\ \alpha_3 &= +(1-2\tau-\sqrt{1-4\tau})/2 \qquad & \alpha_4 &= +(1-2\tau+\sqrt{1-4\tau})/2 \end{aligned} \tag{50}$$

in which $\alpha_3 = \alpha_4$ for $\tau > 1/4$ and the last two line integrals in (48) are merged together.

VI – Discussion and conclusions

Three formulations for second-order drift forces $<F_x, F_y>$ have been presented in the present study. The extension to including the drift moment $<M_z>$ can be achieved following the same procedure. The nearfield formulation based on pressure integrations has been considered as the direct method. However, there are several missing terms in previous studies associated with the integral on the intermittant surface, i.e. the Taylor expansions, in particular, associated with the vertical coordinate, and the inclined angle of hull at the waterline due to rotations. Furthermore, the inclusion of steady wave elevation η_0 and the second-order steady flow $\bar{\psi}$ as in [4], makes the nearfield formulation more general. Numerical application of the nearfield formulation contain some difficulties associated with the higher derivatives of linear potentials and the evaluation of second-order steady potential.

2. this might not be true in general case.

The midfiled formulation is derived based on the principle of momentum conservation for any time-harmonic flow in an enclosed domain with a control surface, with the inclusion of second-order steady flow $\bar{\psi}$. Unlike the nearfield formulation, the wave fields and their higher derivatives can be computed numerically in an accurate way on the control surface at some distance from the ship hull. In principle, the midfield formulation is equivalent to the nearfield formulation although the formal demonstration of their equivalence by using Gauss' and Stokes theorems like zero-speed case shown in [1] is missing.

Putting the control surface at the infinity and using the asymptotic expressions of linear potentials, the farfield formulation can be obtained by the integrals written on the control surface. The pioneer work by Maruo [7] and that by Kashiwagi [5] are just reported here. The farfield formulation (48) is much simpler comparing with the nearfield formulation (26) and the midfield formulation (43). In addition to some simplifications embedded in (47) and in (49) for the Kochin functions, there are some numerical issues raised recently in [9], associated with the infinite integrals in (48). Unlike the case at zero speed where only one wave system (circular ring waves) is present in the far field, there are several wave systems with forward speed including ring waves (upstream and downstream for $\tau < 1/4$) and two downstream V-waves each of which has divergent waves and transverse waves, as shown in [3]. The self-interaction of each wave system is well represented in the farfield formulation (48) while the interaction between different wave systems seems missing.

Implementation of above three formulations in our in-house code HydroStar-V is on-going. The comparison of results obtained from different formulations will be presented in the conference.

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