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# ECOULEMENT INSTATIONNAIRE SUR LE COURANT DEVIE PAR NAVIRE

*(Résumé étendu proposé aux JH2020)*

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**Résumé**

Le problème de tenue à la mer d'un navire animé d'une vitesse d'avance dans la houle est réexaminé dans la présente étude. Suivant la décomposition de l'écoulement total en un écoulement de base et l'écoulement de perturbation contenant une composante stationnaire et une instationnaire, une nouvelle linéarisation cohérente de l'écoulement de perturbation, en particulier, la condition aux limites sur la surface libre, est formulée. En choisissant le courant dévié par le navire (l'écoulement de double modèle) comme l'écoulement de base, un nouveau système d'équations intégrales sur les frontières a été établi en appliquant le théorème de Green. Les équations intégrales comprennent une intégrale localisée sur une zone de surface libre proche du corps, sans l'intégrale le long de la ligne de flottaison qui est présente dans l'approche classique avec la linéarisation sur l'écoulement uniforme. Pour étudier l'intégrale sur la surface libre, la fonction de Green est reformulée par l'introduction de dissipation en raison du viscosité de fluide tel que les comportements singulières et hautement oscillatoires de la fonction de Green sont éliminés. L'intégrale sur la surface libre localisée est donc effectuée sans difficulté majeure. Des résultats numériques montrent que la présente méthode fournit bien un outil fiable et pratique pour étudier les efforts sur un navire animé d'une vitesse d'avance dans la houle ainsi que les réponses dynamiques du navire.

**Summary**

The classical problem of wave radiation and diffraction around a ship advancing in waves is re-considered. Based on the decomposition of total flow into base flow and perturbation flows which contain steady and unsteady components, a consistent linearisation of perturbation flows is carried out. By choosing the ship-shaped stream (double-body flow) as the base flow, a new set of boundary integral equations (BIE) are established by applying the Green's theorem. The resultant BIE includes a localized free-surface integral in the vicinity of ship but without the troublesome waterline integral in the classical Neumann-Kelvin approaches (NK). To treat with the free-surface integral, the classical Green function associated with a pulsating and translating source is modified by considering the dissipation effect so that the complex singular and highly-oscillatory behaviour disappear. The special free-surface integral can thus be evaluated without major difficulty. Numerical results shows that this new method provides the sound and reliable solution to ship seakeeping with forward speed, to evaluate wave loads and induced ship responses.

## 1. Introduction

Being critically important in the design of ships, many studies have been done in the past for studying ship seakeeping with forward speed. No need to mention tremendous progress in applying CFD to ship seakeeping, we like to focus on the classical potential theory derived from the assumption of idea fluid and irrotational flow. There have been two mainstream methods including the NK approach based on the use of Green's function (GFM) and the Rankine source method (RSM). The classical GFM reduces the number of unknowns on the hull and along waterline since it satisfies the boundary condition on the free surface. Major difficulty consists of computing the Green's function, its derivatives and their integration on the hull and along the waterline, as summarized in [1]. On the other side, the RSM is relatively simple to evaluate and accommodating to a variety of boundary conditions on the free surface. However, large number of unknowns have to be distributed over the free surface and uncertain errors associated with the truncation and the design of numerical damping zones impede its applications in practice. Considering advantages and drawbacks of two methods, a multi-domain method has been developed recently in [2], to combine the use of RSM in an interior domain surrounding the ship but limited by a control surface of cylindrical form and the use of GFM in the complementary exterior domain beyond the control surface.

In the present paper, we revisit the GFM with two new developments. First, a consistent linearisation of unsteady flow over the ship-shaped stream, often called double-body flow, leads to BIE including a localised free-surface integral in the vicinity of ship but without the troublesome waterline integral. Second, the classical Green function associated with a pulsating and translating source is modified by introducing the dissipation effect based on the formal analysis of the Laplace-Fourier transform applied to the Stokes flow presented in [4]. The complex singular and highly-oscillatory behaviour in classical Green's function analysed in [3] disappear so that the free-surface integral can be evaluated without major difficulty.

## 2. Ecoulement instationnaire autour d'un navire

We define a Cartesian coordinate system translating at the speed $U$ with the ship in the positive $x$-direction. The $z$-axis is positive upwards with the origin at the undisturbed free surface. Relative to this reference frame, there exists an ambient flow $-U\vec{i}$ opposite to ship forward direction. The presence of ship in this ambient flow creates a ship-shaped steady flow around the hull, called base flow $\mathbf{W} = U\nabla(\bar{\phi} - x)$. In addition to this base flow, there should be a wavy steady flow $\nabla\phi$. When ship oscillates about the reference frame or/and in incoming waves, there exist also unsteady flow $\nabla\psi$. The wavy steady and unsteady flows called perturbation flow represented by the velocity potential $\Phi = \phi + \psi$. The velocity potentials $(\bar{\phi}, \phi, \psi)$ satisfy the Laplace equation in fluid. The total flow $\mathbf{W} + \nabla\Phi$ satisfies the kinematic and dynamic conditions written in the combined form

$$\begin{aligned} \Phi_{tt} + g\Phi_z + 2\mathbf{W}\cdot\nabla\Phi_t + \mathbf{W}\cdot\nabla(\mathbf{W}\cdot\nabla\Phi) + \nabla\Phi\cdot(\mathbf{W}\cdot\nabla)\mathbf{W} \\ = -2\nabla\Phi\cdot\nabla\Phi_t - (\mathbf{W}+\nabla\Phi)\cdot(\nabla\Phi\cdot\nabla)\Phi - \nabla(\mathbf{W}\cdot\nabla\Phi)\cdot\nabla\Phi - g\bar{\phi}_z - \mathbf{W}\cdot(\mathbf{W}\cdot\nabla)\mathbf{W} \end{aligned} \tag{1}$$

on the free surface $z = \eta$ which is defined by

$$\eta = -\frac{1}{g}\left[(\partial_t + \mathbf{W}\cdot\nabla)\Phi + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + \frac{1}{2}(\mathbf{W}\cdot\mathbf{W} - U^2)\right] \tag{2}$$

The above equations (1) for potentials and (2) for wave elevations are fully nonlinear with quadratic and cubic products of potentials since only assumption of time independence concerns the base flow $\mathbf{W}$. Direct solutions of such problems are extremely difficult if not impossible.

### 2.1 Conditions aux limites sur la surface libre

We assume now the base flow $\mathbf{W} = U\nabla(\bar{\phi} - x)$ is of order $O(1)$ while the perturbation flows $\Phi = \phi + \psi$ are of smaller order $o(1)$ comparing to the base flow $(\bar{\phi} - x)$. In this way, the quadratic and cubic products of $(\phi, \psi)$ are ignored. Furthermore, the free-surface elevation $\eta$ is also assumed to be of smaller order $o(1)$ which is true for small or moderate speed. The Taylor expansion of all terms in (1) with respect to $z = 0$ can be obtained by using $T|_{z=\eta} \approx T|_{z=0} + \eta T_z|_{z=0}$ in which $T$ represents any term in (1) and (2). Finally, the frequency-domain expression of unsteady potential is written as $\psi = \Re e\{\varphi e^{-i\omega t}\} L\sqrt{gL}$ and the base flow $\mathbf{W} = U\mathbf{w}$ with $\mathbf{w} = \nabla\bar{\phi} - \vec{i}$ is used to obtain the linear boundary condition

$$\varphi_z - f^2\varphi - 2i\tau\mathbf{w}\cdot\nabla\varphi + F^2\mathbf{w}\cdot\nabla(\mathbf{w}\cdot\nabla\varphi) + F^2\nabla\varphi\cdot(\mathbf{w}\cdot\nabla)\mathbf{w} + \bar{\phi}_{zz}(i\tau\varphi - F^2\mathbf{w}\cdot\nabla\varphi) = 0 \tag{3}$$

on $z = 0$. In (3), we have used the notations $f = \omega\sqrt{L/g}$ for encounter frequency, $F = U/\sqrt{gL}$ the Froude number and $\tau = fF$ the Brard number with $L$ the ship length.

### 2.2 Fonction de Green avec dissipation

We define the fundamental solution at the field point $P(x_p, y_p, z_p)$ associated with a translating and pulsating source located at $Q(x, y, z)$, i.e. the Green's function $G(P,Q)$ which satisfies the special equation of Poisson type $\nabla^2 G(P,Q) = 4\pi\delta(|P-Q|)$ with $\delta(\cdot)$ the Dirac function. Based on the formal analysis of the Laplace-Fourier transform applied to the Stokes flow, in [4], the leading effect of vorticity is represented by an additional term appearing in the boundary condition at the free surface. Indeed, the linear boundary condition with dissipation is written by

$$G_z - f^2 G - 2i\tau G_x + F^2 G_{xx} + 4\varepsilon(F G_{xzz} - if G_{zz}) = 0 \tag{4}$$

on $z = 0$. In (4), the coefficient $\varepsilon = \mu/(\rho\sqrt{gL^3})$ is proportional to the fluid viscosity $\mu$. It is shown that the magnitude of elementary waves $e^{kz+i(kx-\omega t)}$ decays like $e^{-4\varepsilon\omega k^2 x}$ more rapidly with short waves of large wavenumber. This implies that the complex singular and highly oscillatory behaviours in $G(P,Q)$ due to short waves predicted in [3] just disappear.

### 2.3 Equations intégrales aux frontières

Applying the Green's theorem to the couple potentials $(\varphi, G)$ in the fluid domain and on all the boundaries including the hull $H$, the free surface $S$ and at infinity $S_\infty$, we have the integral equation

$$2\pi\varphi^H + \oint\!\!\oint_H \varphi^H G_n \,ds + \iint_S \varphi^F (G_z - f^2 G)\bar{\phi}_x \,ds = \iint_H \varphi_n^H G \,ds \tag{5}$$

for $P \in H$ and that

$$4\pi\varphi^F + \iint_H \varphi^H G_n \,ds + \oint\!\!\oint_S \varphi^F (G_z - f^2 G)\bar{\phi}_x \,ds = \iint_H \varphi_n^H G \,ds \tag{6}$$

for $P \in S$. The integral on $S_\infty$ is nil due to the radiation conditions for $(\varphi, G)$. The integral on the free surface $S$ involving the boundary condition (3) for $\varphi$ and (4) for $G$ is analysed in a similar way as [5] but different results are obtained. In fact, the free-surface integral is partially transformed into waterline integrals by making use of Stokes theorem. The integrand function of waterline integrals contains the factor $\mathbf{w}\cdot\mathbf{n}$ which is zero if the ship-shaped stream $\mathbf{w}\cdot\mathbf{n} = 0$ on $H$ and the condition $\bar{\phi}_z = 0$ on $z = 0$ are imposed. The remaining part of free-surface integrals is dominated by the third term on the left side of (5) and (6). This term is significant only in the vicinity of waterline due to fast decay of double-body flow $\bar{\phi}_x$.

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