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A DISCUSSION ON THE WAVE BREAKING CRITERIA OF SHALLOW WATER OCEAN WAVES

UNE DISCUSSION SUR LE CRITERE DE DEFERLEMENT DES VAGUES EN EAU PEU PROFONDE

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Summary

Accurate prediction of wave breaking in the coastal zone is important for various engineering activities and environmental issues. Numerous studies have been undertaken to describe when and where wave breaking occurs. However, there is no universal formulation so far to account for wave breaking in numerical models such as Boussinesq-type models. The recent study of [3] in intermediate and deep water defines an energetic criterion that seems promising at providing a more universal criteria. In the present paper, we assess the applicability of this breaking criteria in shallow water, based on a 2D fully nonlinear potential model.

Résumé

Prédire de manière précise le déferlement des vagues en zone côtière est important pour diverses activités d'ingénierie et de problèmes environnementaux. De nombreuses études ont été réalisées pour décrire quand et où se produit le déferlement des vagues. Cependant, il n'existe pas encore de formulation universelle permettant de prendre en compte le déferlement des vagues dans les modèles numériques tels que les modèles de type Boussinesq. La récente étude de [3] en eaux intermédiaires et en eaux profondes définit un critère énergétique qui semble prometteur pour fournir un critère plus universel. Dans cet article, nous évaluons l'applicabilité de ce critère de déferlement en eaux peu profondes, à partir d'un modèle potentiel non-linaire 2D.

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I – Introduction

Breaking waves create significant dynamical loadings on ocean engineering structures. Waves at the transition to breaking are critical design condition for marine and coastal structures. Optimal structural design require an accurate prediction of the onset and strength of wave breaking in the surf zone. Coastal engineering studies also need detailed wave characteristics at the breaking point [6, 14]. The surf zone dynamic is very complex, many processes and physical phenomenon (shoaling, refraction, diffraction, infragravity waves, rip currents, wave breaking) that happen at different time and space scales interact. The nonlinear and irregular processes that occur just before a wave breaks, make predicting its onset difficult.

A significant effort has been undertaken to accurately simulate nonlinear wave transformations towards breaking in shallow water in the past few decades for various nearshore activities and environmental issues. The choice of a proper numerical model to simulate the wave evolution toward breaking are among the important issues that must be considered. Numerical models such as Boussinesq-type (BT) models assume a single-valued free surface and can not reproduces the crest overturning inherent to breaking waves, therefore they can not reproduce breaking waves. To solve for wave breaking in BT models, two steps are required : a wave-breaking onset criteria and a method for computing wave breaking energy dissipation.

Many wave breaking onset criteria, aimed at practical applications, have been proposed over the last half century through theoretical study, numerical simulations, laboratory experiments or field observations [30, 12]. These criteria can be classified into three categories : geometric, kinematic and dynamic.

I – 1 Geometric breaking criteria

Geometric criteria use either a steepness threshold, a wave asymmetry threshold or an angle of the wave front threshold to determine wave breaking onset. This type of breaking criteria has been used as indicator of breaking onset in deep and shallow waters [20, 1, 23, 15]. As described in the wave breaking review of [19] in deep water, the threshold value can vary widely depending on the breaker type and the method used to generate the breaking event. In shallow water, [23, 15, 26, 5, 31] use different front surface angles threshold for wave breaking onset and termination in BT models. These thresholds have to be calibrated depending on the bathymetry and the breaker type.

Such differences in the wave geometry at breaking make the geometric criteria unsuitable as a breaking onset criteria in deep and shallow waters.

I – 2 Kinematic breaking criteria

The deep water kinematic breaking criterion is based on the relationship between the horizontal wave crest particle velocity u and the wave phase speed c. Wave breaking occurs when $u/c \geq 1$. Difficulty arises in this criterion when determining u and c for highly unsteady and rapidly changing waves. From experiments in deep water cases, [27, 34, 2, 30] found different threshold values of the ratio u/c depending on the breaking configurations (type of breaking wave, wave generation mechanism, etc). In shallow water, the breaking criteria of [13, 32, 6] used in BT models can be classified as kinematic criteria. [13] defines a criterion based on the normal speed of the free surface elevation, [32] uses a relative Froude number and [6] combines the two previous criteria. These criteria have been used in other studies [4, 17] that provide different threshold values of the calibration parameters.

Although the kinematic or geometric wave properties have been traditionally used for wave breaking criteria in deep or shallow waters, these criteria require calibration and are not universally applicable.

I − 3 Dynamic breaking criteria

Empirical findings based on observations of modulated deep water wave groups suggest that breaking occurs when the local flux of energy in a wave, generally close to the center of the group, exceeds a given threshold ([25, 29]).

[3] have recently proposed a wave breaking threshold parameter B based on the local energy flux relative to the local energy density, normalised by the local crest speed as described in the following equations.

$$\mathbf{B} = \frac{\mathbf{F}}{E\|\mathbf{c}\|} \tag{1}$$

with E the energy density defined as

$$E = \rho g(z - z_0) + \frac{1}{2}\rho\|\mathbf{u}\|^2 \tag{2}$$

and $\mathbf{F}$ the local energy flux defined as

$$\mathbf{F} = \mathbf{u}\left((p - p_0) + \rho g(z - z_0) + \frac{1}{2}\rho\|u\|^2\right) \tag{3}$$

where p is the pressure, p_0 is the pressure above the surface, $\|\mathbf{u}\|$ is the fluid speed, g is gravitational acceleration, z is the vertical coordinate and z_0 is the vertical level of reference.

With a zero surface pressure condition, the parameter defined by [3] reduces to B_x, the ratio of the surface fluid speed u in the wave propagation direction (x) to the crest point speed c :

$$B_x = \frac{F_x}{Ec_x} = \frac{u_x}{c_x} \tag{4}$$

According to numerical simulations of fully-nonlinear 2D and 3D wave packets in deep and intermediate water depth, when B_x exceeds the value 0.85, the wave will inevitably undergo breaking onset. If the crest fluid speed of a wave does not exceed 0.85 of its crest speed, the wave will not break. This threshold value has been validated with the experiments of [22]. [24, 22, 21] investigated the breaking onset of [3] against laboratory measurements. They found the onset of breaking threshold to be robust for different types of wave groups in deep and intermediate water. This approach seems to be promising at providing a more universal criteria in deep and intermediate water. It has not been applied to shallow water conditions though [3] suggests that it should be valid. This would mean that even though the processes leading to wave breaking are different in deep (wave group modulation) and shallow water (wave shoaling), the concept of energy flux rate threshold could be valid in any water depth. This is precisely the question addressed in the present study, we would like to assess the applicability in shallow water of the breaking criteria introduced by [3]. For this, we shall use the fully nonlinear potential flow (FNPF) model developped by [10] to analyse solitary breaking waves in 2D.

The paper starts with the description of solitary wave breaking database generated from the FNPF model. Preliminary results regarding the wave breaking threshold are presented. The paper ends with a discussion regarding this study.

II – Methodology

II – 1 The FNPF model

FNPF model represents 'numerical wave tank' in which numerical experiments can be set up and used to gain physical insight into complex wave phenomena. The FNPF model is based on fully nonlinear potential flow theory and combines a higher-order boundary element method (BEM) for solving Laplace's equation at a given time and Lagrangian Taylor expansions for the time updating of the free surface position and potential. Potential flow theory can quite well predict the physics of wave shoaling over a slope, up to and into the early stages of breaking, before touchdown of the breaker jet on the free surface. BEM techniques are efficient for representing wave propagation and overturning until the wave surface reconnects [11]. [8] provides extensive validation of shoaling and breaking solitary waves cases against laboratory data. The FNPF model of [10] provides accurate predictions of height and location of breaking, and detailed characteristics of waves at the breaking point can be determined. Because of the accurate comparison between the model and the measured data, the FNPF model can be used as a reference to investigate detailed characteristics of breaking waves. [33] uses the FNPF model as a standard of accuracy. The results from this code are used to assess the applicability in shallow water of the breaking criteria introduced by [3].

II – 2 Input wave

Solitary waves are often used as a simple model for studying propagation, shoaling, and breaking of extreme waves in shallow water. Solitary waves closely model tsunamis and can also be used to represent surf-zone waves [8, 9]. They are simple to deal with in a FNPF model. Observations suggest that waves approaching a beach often resemble solitary waves [18]. We are studying wave breaking generated by shoaling over a gentle plane slope (Figure 1). Numerically exact solitary waves (obtained from the fully nonlinear method by [28]) are used as incident waves in the constant depth region prior to the slope.

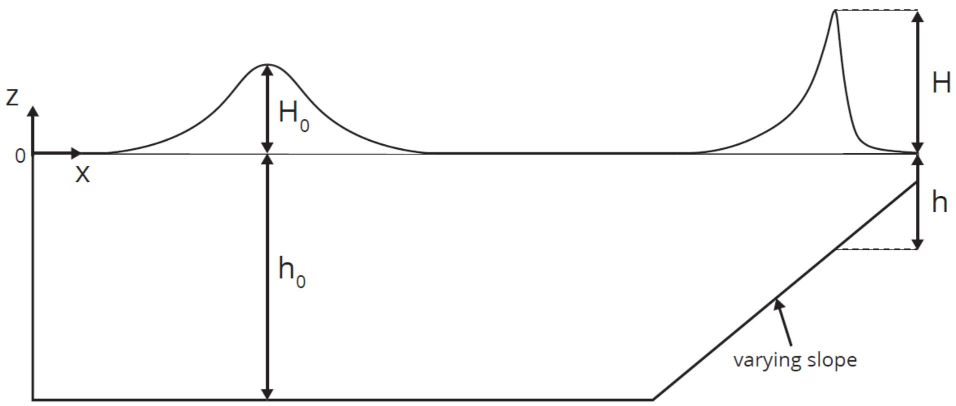

Figure 1 – Definition sketch of the numerical experiments with the FNPF model.

II – 3 Computational domain

The numerical domain used to generate breaking waves is represented on Figure 1. The depth of the flat region is $h_0 = 0.30m$. The plane slope is varied from 2% to 12%. Solitary waves of initial wave height $H_0/h_0 = 0.5$ to 0.67 are propagating in the domain. By combining the different initial wave heights and slopes, we were able to generate more than 50 cases of solitary breaking wave.

There are 461 nodes in the discretization, including 300 nodes on the free surface (with initial spacing $\Delta x = 0.117m$). The distance between nodes on the bottom is constant over the constant depth region and this distance is reduced over the slope to increase the resolution when depth decreases. On the surface, the nodes converge toward the crest region during shoaling while the distance between nodes increases in the front and in the back of the waves.

The breaking point is arbitrary defined as the crest location for which the free surface slope becomes vertical in the wave front. Shortly after the slope exceeds the vertical, numerical errors increase leading to instability of computations and the model blows up.

Figure 2 shows the results for a solitary wave of initial height $H_0 = 0.17m$.

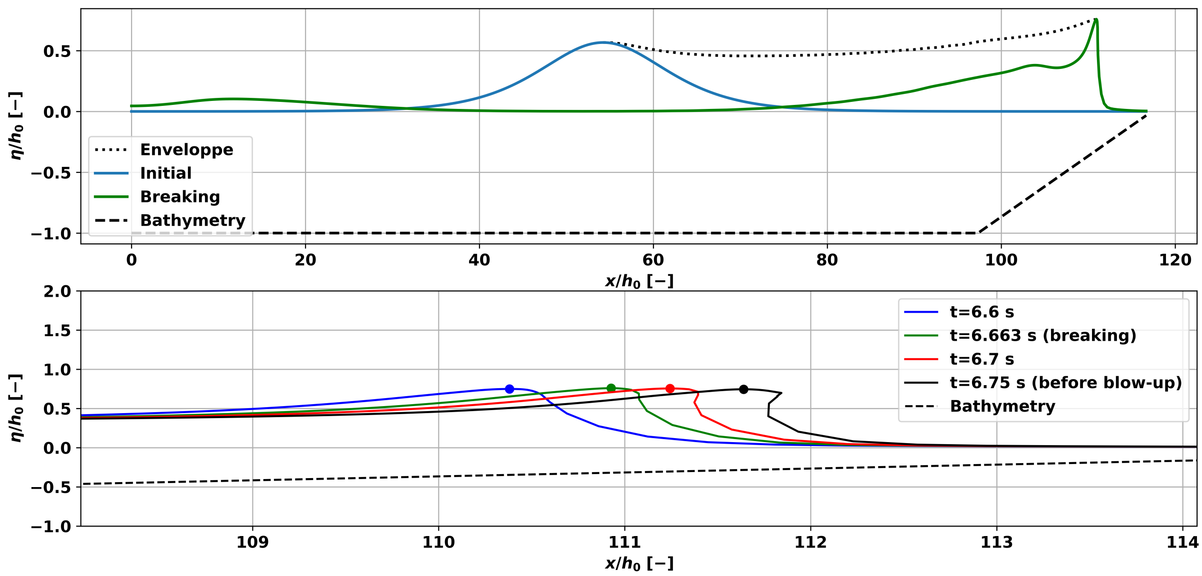

Figure 2 – Solitary wave breaking over a 5% slope from the FNPF model (with an initial soliton height of H_0=0.17m.

III – Results

We are investigating the applicability of B_x parameter in shallow water. Let's define B_{SW} the equivalent of B_x in shallow water for the present configurations. To determine the breaking parameter B_{SW} at the free surface, at every point in the domain and at every time step, we need to compute for every breaking cases, the ratio between the horizontal fluid velocity at the crest (u_c) and the phase speed c according to [3]. The crest is defined here as the maximum surface elevation at each time step.

We performed a spatial cubic interpolation on the elevation and potential velocity variables to obtain a better refinement of the crest until the breaking point. Also because the temporal resolution vary at each time step, we interpolated on a regular time vector.

The horizontal fluid velocity at the crest (u_c) is computed from the velocity potential ϕ, the working variable of the FNPF model, based on the potential theory :

$$u_c = \frac{d\phi}{dx} \tag{5}$$

To compute B_{SW}, this requires the phase velocity to be calculated instantaneously everywhere in the domain and at every time step. At the location of the crest, the phase speed is the crest velocity. The crest velocity c is then calculated based on a simple crest tracking method : from the shift in time of the maximum surface elevation, we are able to compute at each time step the ratio between the distance the crest has traveled (Δx) over the time required to travel this distance (Δt) :

$$c = \frac{\Delta x}{\Delta t} \tag{6}$$

Figure 3 presents for every cases the value taken by B_{SW} at the moment of breaking against the parameter S_0 which is the surf similarity parameter of solitary wave defined in [8] :

$$S_0 = 1.521 \frac{s}{\sqrt{H_0/h_0}} \tag{7}$$

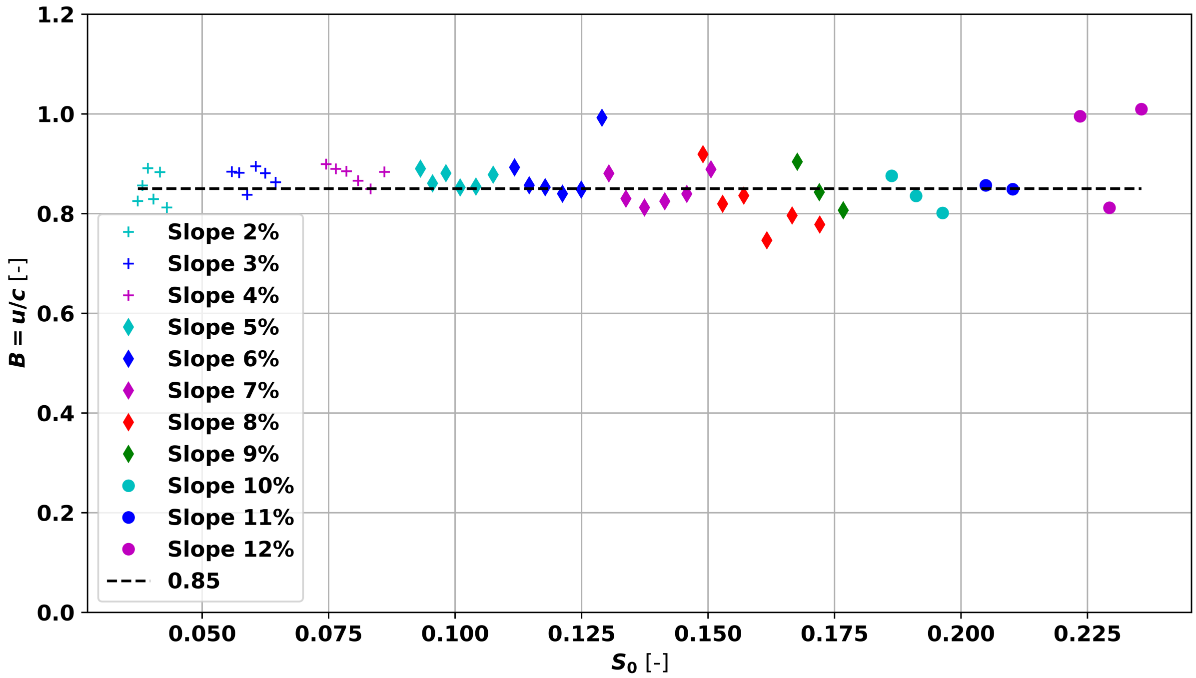

Figure 3 – $B_{SW} = f(S_0)$ at breaking point for different slope and height with the FNPF model. S_0 is the surf similarity parameter of solitary wave defined in [8] : $S_0 = 1.521 s/\sqrt{H_0/h_0}$.

According to the S_0 values of each simulations, all the waves generated are of plunging type. Our preliminary results show that most wave breaking occurs above the threshold $B_{SW} = 0.85$ which is in agreement with Barthelemy's paper where they report that any waves with a B_x above 0.85 will inevitably and quickly (a fraction of period) evolves toward breaking. Some waves break at lower values than $B_{SW} = 0.85$. Waves that break before $B_{SW} = 0.85$ mostly propagates on the mildest slopes (between 7% and 10%). The mean B_{SW} is 0.86. 95% of the waves break above $B_{SW} = 0.80$ and 85% of the waves break with B_{SW} within the range 0.80-0.90.

We note that the dispersion around the mean B_{SW} value could be due to uncertainty on the phase speed calculation. The simple procedure we used might lead to errors in the determination of the crest velocity and constitutes a difficulty in the reliable verification of this criterion. The time and spatial resolutions might not be sufficient to capture the highly nonlinear and rapid process of wave breaking.

Figure 4 shows the values of B_{SW} plotted against the time for every cases. We notice on this figure that the evolution of B_{SW} with time strongly varies depending on the configuration. Cases with high B_{SW} values at breaking (around 1) reach such rates very rapidly. On the contrary, it seems that many cases that break with low B_{SW} values (cases corresponding to the mildest slopes), have a slower evolution of the B_{SW} parameter before breaking. We can also see that for some cases, B_{SW} is not continuously increasing and decreases a short time before breaking. This might be because the exact time of breaking is not correctly captured. This suggest, that the exact time of breaking might not be captured with a sufficient accuracy. Improving our detection method, with e.g. a better time and spatial resolution would probably lead to a smaller dispersion in the B_{SW} values.

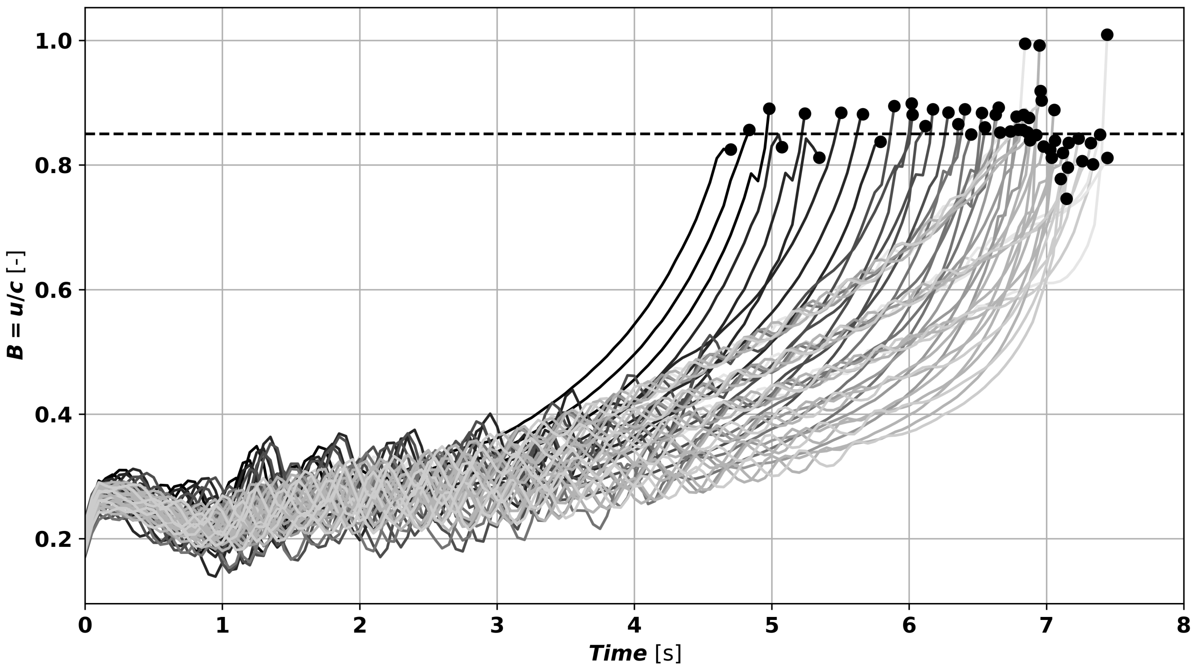

Figure 4 – B_{SW} against time. The darkest line is for the lowest S_0. S_0 increases with the line becoming lighter.

IV – Conclusion

The threshold $B_x = 0.85$ defined by [3] allows to detect wave breaking within a very short time before it actually happen (up to a fifth of a carrier wave period prior to a breaking event) for deep and intermediate water. This criterion can be assimilated to a "point of no return" for wave breaking. Our preliminary results suggest that the equivalent "point of no return" in shallow water is slightly lower than the one identified in deep and intermediate water. The large majority of the waves break above $B_{SW} = 0.80$.

It is also noticed that on the contrary to the results of [3] (figure 6), B_{SW} values at breaking are concentrated around their mean value. The dispersion around the mean B_{SW} is small and could be reduced by increasing the spatial and temporal resolutions. This suggests that the mean B_{SW} (very close to $B_x = 0.85$) could be used as an actual wave breaking criterion in shallow water and not as a threshold for predicting wave breaking as in deep and intermediate water.

It should be stressed that the stability of BEM simulations close to breaking strongly depends on the parameters of the numerical scheme. Further effort therefore need to be invested to increase the resolution to obtain a more accurate wave breaking onset threshold in shallow water conditions over a larger type of breaking waves (surging, plunging and spilling). Also the present simulations are in 2D. We would like to extend this study to 3D simulations.

This study is a first step towards an improved breaking threshold to be used in shallow-water phase-resolving models. However, further efforts are needed to obtain a more accurate value of B_{SW}.

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