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CARACTERISATION DES EFFORTS DU SECOND ORDRE EN FAIBLE PROFONDEUR AVEC UN FOND INCLINE

SECOND ORDER LOADS CHARACTERIZATION IN SHALLOW WATER WITH A SLOPE BOTTOM

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Summary

An experimental campaign was previously reported on the slow-drift motion of a rectangular barge moored in irregular beam seas. The 24 m long false bottom of the basin is raised and inclined at a slope of 5% from 1.05m below the free surface to 0.15m above (see figure 1). The barge is moored in different water depths (from 54 cm to 21 cm). The measured slow-drift component of the sway motion is compared with state of the art calculations based on Newman approximation. The principal results are, for the beam sea configuration:

- at 54 cm depth: good agreement between the experimental results and the Newman simulation
- at 21 cm depth: the Newman calculation overpredicts the forces.

When the flat bottom set-down contribution is added, the calculated value is much larger than the measured one. It has often been observed that the flat bottom expression of this component leads to over-conservative second order loads. A second order model is then proposed to account for the shoaling effect of a bi-chromatic sea-state propagating in decreasing water depth.

Newman's approximation provides an estimate of the real part P. Q is mostly a contribution of the second order incident part in flat bottom. When $R < 1$ and $0 < \alpha < \pi/2$ the modulus of the QTF is decreased as compared to its flat bottom reference value [8].

Application to the model tests shows that, due to shoaling, the set down contribution to the slow drift excitation can subtract and not add up to the Newman component .

A second experimental campaign test is made with the same slope bottom configuration but with the barge moored successively in different locations and positions with respect to the wave (head, beam and 30°).

The previous second order model is adapted to take into account the difference of depth between the front of the barge and the aft of the barge in the DIODORE™ software.

For the head sea cases, the Newman approximations under-estimate the low frequency motions, and the correction of the set down gives better comparisons with the measured values.

In parallel to the development of the set-down modified method, a numerical model based on the Boussinesq formulation with high order is tested to extract the different components of the generated wave on the slope bottom. The tests studied from the campaign in BGO First basin are regular waves, bi-chromatic, bi-chromatic+irregular. Analysis of the difference frequency component and the set down contribution are emphasized with comparison with semi-analytical solutions.

I – Introduction

There has been an increasing interest lately for marine operations in shallow water; examples are nearshore pipe-laying and LNG terminals. This interest is closely related to the expected development of LNG consumption and to ongoing projects of LNG terminal and LNG floating storage units. Typically these offloadings terminal or FSRU (Floating Storage Regasification units) would be located at 15 m to 30 m of water depth, away from the coast, in exposed locations.

There are many associated hydrodynamic issues, one of them being the prediction of the wave induced mooring loads, a second one is the characterization of the components of the incident waves, currently taken as a Stokes model.

As they travel shorewards, ocean waves undergo various transformations over the decreasing bathymetry, the most relevant one, for the problem considered here, being a transfer of energy from the primary waves to the accompanying long waves. Also known as set-down or, in coastal areas, as infragravity waves, these long waves scale with the wave envelope signal.

According to its flat bottom expression [7], the set-down contribution to the second order loading increases as the water depth decreases. As a result Newman's approximations become unapplicable and some account must be given to the set-down contribution to the low frequency loading.

In section 2, we describe an experimental campaign, carried out at the BGO-First basin in La Seyne sur Mer. A false bottom is raised and inclined so as to achieve a beach, over 20 m long inclined with a 5% slope and starting with a depth of 1.05 m. A rectangular barge is moored at different locations and submitted to long crested irregular beam, head and 30°heading seas. The standard deviation of its measured slow-drift sway and surge motion is compared with state of the art calculations. Depending of the configurations, it is found that the calculations over-predict measurements (beam seas) or under-predict measurements (head seas).

In section 3, we address the problem of the shoaling of the long wave associated with a bi-chromatic wave system. A theoretical model is proposed where the varying bathymetry is decomposed as a succession of steps, defining rectangular subdomains where the second-order problem can easily be solved.

Finally, in section 4, we present the results obtained with the computations of the slow-drift motion for different dof (degree of freedom) made with the DIODORE^{TM} software, upgraded with the formulation proposed in section 3. The up-dated values of the slow-drift motion are in good agreement with the experimental values.

II – Experimental campaign

The experiments were performed in the BGO-First offshore tank in La Seyne sur Mer. This basin has a total length over 40m and a width of 16m. Thanks to a false bottom, the water depth can be varied between 5m and a few centimeters. In these experiments, the 24m false bottom was both raised and inclined at a slope of 5%, starting from a depth of 1.05m by the wavemaker side and emerging at 15cm at the other end. Figure 1 shows a sketch of the setup.

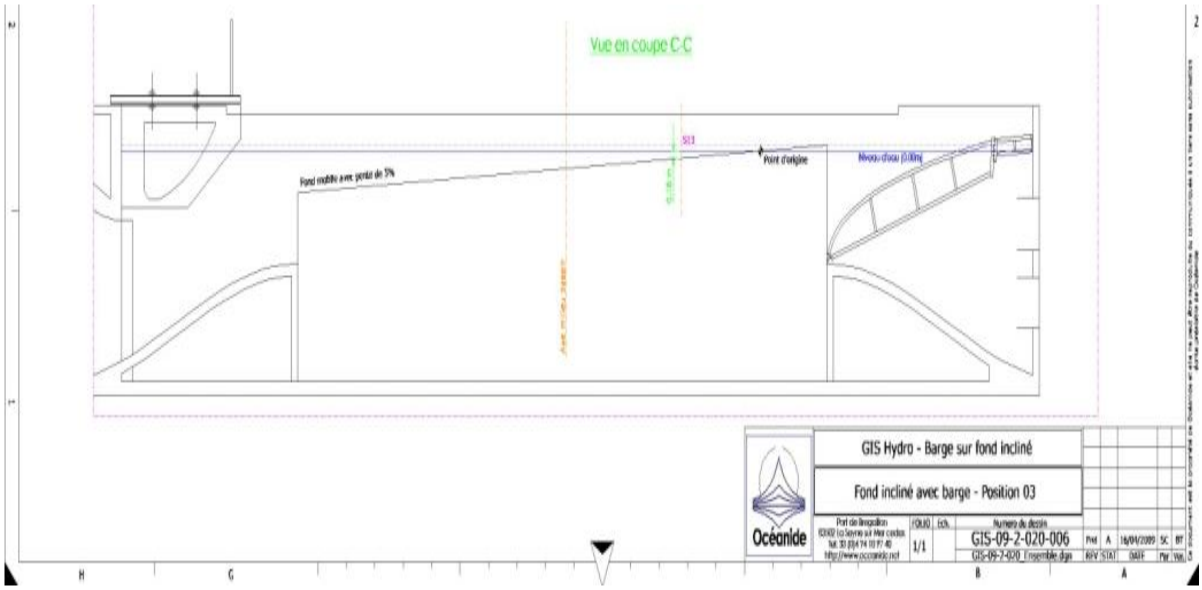

Figure 1 : Experimental campaign configuration

The rectangular barge model as described in [5] was used for the tests. This model has a length of 2.47 m, a beam of 0.60 m, a depth of 0.30 m and square bilges. Its draft during the tests was 0.12 m, with the center of gravity 0.135 m above the keel line and a roll radius of gyration of 0.19 m. Figure 2 shows a picture of the model undergoing tests at an intermediate position and with a 30° heading from the wave direction.

The mooring stiffness was chosen in order to yield a surge, sway and yaw natural period around 8.5s. Because of the sensitivity of the dof (degree of freedom) added mass to the keel clearance, the mooring stiffness is adjusted to reach always the same natural period (from 8.2s to 8.6s). The retained scale factor is 100.

The barge model was submitted to irregular waves of Pierson Moskovitz spectra with peak periods of 1.2 s and 1.6 s. The associated Hs, specified and calibrated with the horizontal false bottom at 1.05m depth, are:

- Tp=1.2 s : Hs = 2 cm, 4 cm, 6 cm;
- Tp=1.6 s : Hs = 2.5 cm, 5 cm, 7.5 cm.

During the calibrated tests, the reflection coefficients were found to be less than 2% in all wave conditions.

The water depths at the barge locations are 54 cm, 29 cm and 21 cm.

The barge motion was measured with the optical system Krypton-Rodym which ensures an accuracy better than 1 mm. Test duration in irregular waves was 1200 seconds, meaning 140 low frequency dof cycles. Through low-pass filtering, the dof slow drift motions were extracted from the time series and the standard deviation derived.

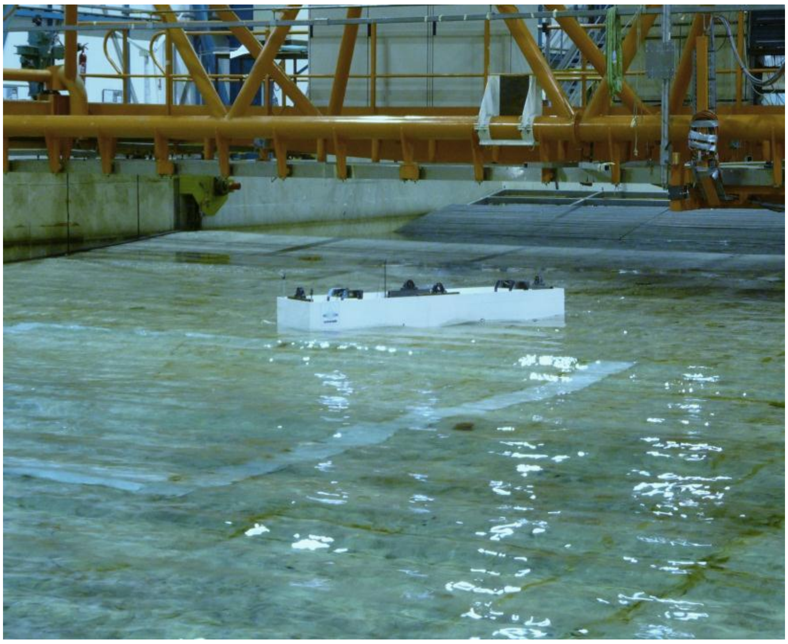

Figure 2 : Barge model undergoing irregular wave tests at the shallowest position

III – Comparison of the measured slow drift motion with different approaches

Drift forces were calculated with the DIODORE™ software of PRINCIPIA at different water depths, assuming a flat bottom. The Lagally formulation was used [6]. In the calculations, a quadratic damping moment (accounting for flow separation at the bilge) in roll was introduced through stochastic linearization [5].

A comparison was made between Newman approximation, complete quadratic transfer function (QTF) matrix based on a Lagally approach [12] and measured slow drift motion. It was shown that to better approximate the results of the experimental tests, the shoaling of the set-down has to be taken into account [8] [11].

An explanation of the different terms is given below.

The Quadratic Transfer Function of the slowly varying second order force is approximately evaluated by:

$$f_{-}^{(2)}\left(\omega_i,\omega_j\right) = P\left(\omega_i,\omega_j\right) + iQ\left(\omega_i,\omega_j\right) \qquad (1)$$

With P the real part, that can be estimated through the Newman approximation

$$P\left(\omega_i,\omega_j\right) \sim \sqrt{f_d(\omega_i) f_d(\omega_i)} \qquad (2)$$

and Q the imaginary part, mainly coming from the set down.

The definition of the term Q, set down part, is given in [8].

The first standard deviation of the slow drift sway motion (normalized by the significant wave height squared) from measurements and from calculations showed that the values computed with the Newman approximation and the added of the set down term over-estimated the loads. (previous results)

Tp=1.6s, Hs=2.5cm	h=54cm	h=29cm	h=21cm
Y/H_s^2 measured (m^{-1})	15.8	12.3	11.3
Y/H_s^2 computed Newman(m^{-1})	13	14.2	17.6
Y/H_s^2 computed Total(m^{-1})	13.6	20.2	35.8

Tp=1.6s, Hs=5.0cm	h=54cm	h=29cm	h=21cm
Y/H_s^2 measured (m^{-1})	15.8	12.3	11.3
Y/H_s^2 computed Newman(m^{-1})	13	14.2	17.6
Y/H_s^2 computed Total(m^{-1})	13.6	20.2	35.8

Tp=1.6s, Hs=7.5cm	h=54cm	h=29cm	h=21cm
Y/H_s^2 measured (m^{-1})	15.8	12.3	11.3
Y/H_s^2 computed Newman(m^{-1})	13	14.2	17.6
Y/H_s^2 computed Total(m^{-1})	13.6	20.2	35.8

Table 1: Standard deviation of the slow drift sway motion (normalized by the significant wave height squared) with the previous test (sway natural period around 10s)

To approach the measured values, the shoaling of the set-down is taken into account in the second order equation. A more detailed bibliography of the transformation of the long waves in the coastal zone can be found in [8].
The development is made for the two dimensional case of an unidirectional bi-chromatic wave group propagating in the normal direction to a bathymetry with parallel depth contours. The varying depth zone is assumed to be of mild slope and confined in-between two semi-infinite regions of constant depths.
As the bi-chromatic wave group enters the varying depth zone, its primary wave components at frequencies ω_1 and ω_2 are assumed to shoal according to the ray theory. According to [1], unless the bottom slope is very small, the accompanying long wave does not amplify to the extent predicted by the flat bottom theory, and its phase relationship with the wave envelope deviates from the flat bottom value. This means that the Q component of the QTF equation (1) is corrected by some complex factor $R(\omega_1,\omega_2)\exp[i\alpha((\omega_1,\omega_2)]$ so that the QTF becomes

$$f_{-}^{(2)}\left(\omega_i,\omega_j\right) = P\left(\omega_i,\omega_j\right) + iQ\left(\omega_i,\omega_j\right) \times R\left(\omega_i,\omega_j\right) e^{ia\left(\omega_{i,}\omega_j\right)}$$
$$f_{-}^{(2)}\left(\omega_i,\omega_j\right) = P - QRsin\alpha + iQRcos\alpha \qquad (3)$$

A numerical model is the needed that is sufficiently easy and fast to run where R and α can be computed for all couples $\left(\omega_i,\omega_j\right)$ of the wave signal. In the section III.1, such a model is presented, based on a second order step model.

III – 1 Second order step model

The considered bathymetry is two dimensional (parallel depth lines), in-between two semi-infinite domains of constant depths h_L and h_R. The bottom slope is assumed to be mild, so that ray theory can be used to predict the wave transformation, at first order. The first order velocity potential is therefore given by

$$\Phi^{(1)}(x,y,z,t) = \mathcal{R}\left\{\sum_{i=1}^{2} -i\,\frac{A_i(x)g}{\omega_i}\,\frac{\cosh\,k_i(x)[z+h(x)]}{\cosh\,k_i(x)h(x)}\;e^{i\int_0^{-\infty}\nu_i(\chi)d\chi}\;e^{ik_{i0}y\sin\beta_i - i\omega_i t}\right\} \qquad (4)$$

Here x = 0 is the beginning of the varying depth zone, β_1 and β_2 are the initial propagation angles with respect to the x axis (β=0 means normal incidence), k_i and ν_i are the local wave numbers defined by

$$\omega_i^2 = gk_i(x)\tanh k_i(x)h(x) \qquad \nu_i\,(x) = \sqrt{k_i^2(x) - k_{i0}^2\, sin^2\,\beta_i} \tag{5}$$

k_{i0} is the wave number in the upwave sub-domain and the amplitude A_i varies according to (e.g. [10], chapter 3)

$$A_i(x) = A_{i0}\sqrt{\frac{C_{G_{i0}}}{C_{G_i}(x)}\,\frac{k_i(x)\cos\beta_i}{\nu_i(x)}} \tag{6}$$

with C_{Gi} the group velocity.
The second order velocity potential, at the difference frequency $\Omega = \omega_1 - \omega_2$, takes the form

$$\Phi^{(2)}(x,y,z,t) = A_{10}\,A_{20}\,\Re\{\varphi^2(x,z)e^{iK_y y}e^{-i\Omega t}\} \tag{7}$$

with $K_y=k_{10}\sin\beta_1 - k_{20}\sin\beta_2$

$\varphi^2(x,z)$ satisfies the free surface equation (at z=0)

$$g\varphi_z^{(2)} - \Omega^2\varphi^{(2)} = i\,\Omega\,(\varphi_{1x}\varphi_{2x}^* + \varphi_{1z}\varphi_{2z}^* + k_{110}k_{210}\sin\beta_1\ \sin\beta_2\,\varphi_1\varphi_2^*) + \frac{1}{2g}\{-i\omega_1\varphi_1(-\omega_2^2\varphi_{2z}^* + g\varphi_{2zz}^*) + i\omega_2\varphi_2^*(-\omega_1^2\varphi_{1x} + g\varphi_{1zz})\} \tag{8}$$

At z=0, together with the Helmholtz equation

$$\varphi_{xx}^{(2)} + \varphi_{zz}^{(2)} - K_y^2\varphi^{(2)} = 0 \tag{9}$$

in the fluid domain. The no-flow condition on the sea bottom and appropriate matching conditions at the upwave and downwave boundaries of the varying depth zone are then added. The varying depth zone is then decomposed as a series of horizontal steps, following the technique that has been used by many authors to solve the first-order problem (see for instance Rey et al. 1992 or Bender & Dean 2003). In each rectangular sub-domain j, of depth h_j and extending from $A_j(x-x_{j-1})+B_j$. A particular solution, satisfying the non-homogeneous free surface condition, to the second-order problem is then

$$\varphi_{P_j}^{(2)}(x,z) = \frac{A_j\left(x - x_{j-1}\right) + B_j}{gK_y\tanh K_y h_j - \Omega^2}\,\frac{\cosh K_y\left(z + h_j\right)}{\cosh K_y h_j} \tag{10}$$

The general solution is obtained by adding propagative and evanescent modes satisfying the homogeneous free surface equation

$$\varphi_{F_j}^{(2)}(x,z) = \frac{\cosh K_{j0}\left(z + h_j\right)}{\cosh K_{j0}h_j}\left[B_{j0}e^{i\mu_{i0}(x-x_{j-1})} + C_{j0}e^{i\mu_{j0}(x-x_j)}\right] + \sum_{m=1}^{\infty}\cos K_{jm}\,(x + h_j)\left[B_{jm}\,e^{i\mu_{im}\,(x-x_{j-1})} + C_{jm}\,e^{i\mu_{jm}\,(x-x_j)}\right] \tag{11}$$

where the wave number K_{jm} and μ_{jm} verify

$$\Omega^2 = gK_{j0} \tanh K_{j0} \tanh K_{j0} \; h_j = -gK_{jm} \tanh K_{jm} \; h_j \tag{12}$$

$$\mu_{j0}^2 = K_{j0}^2 - K_y^2 \quad \mu_{jm}^2 = K_{jm}^2 - K_y^2 \tag{13}$$

It must be noted that the μ_{j0} coefficients are not necessarily real. In the present paper, we only consider normal incidence where $\beta_1=\beta_2=0$ and $K_y = 0$.
The solution is finally obtained by matching the total second order potentials $\varphi_{Pj}^{(2)}+ \varphi_{Fj}^{(2)}$ and their x-derivatives at the successive boundaries, and stating that there are only outgoing propagative and evanescent modes at the upwave and downwave boundaries at the variable depth zone, superimposed with the locked potentials given by equation of the second order potential in flat bottom. See [7] for details.
In the practical cases that we are interested in the water depth is shallow for the second-order subharmonic wave. This means that the evanescent modes in the expansion (11) can be neglected. The problem can then be solved in two steps: an ”incident” step where the incoming second-order wave is propagated over the variable depth zone, and a “reflection” step which requires ad hoc conditions to be formulated at the end of the variable depth zone. These conditions present no problem when the waterd epth of the downwave semi-infinite region is sufficiently large that the waves do not break. In the case of the experiments in the BGO first basin, there is no constant depth down wave region, the wave break and the incoming subharmonic wave is partly reflected. The mean to treat this term is explained in [7]. In our case, we have chosen to omit the reflection step, too complicate to introduce in a simple model and, whereas there is a well-defined phase relationship between the short wave envelope and the incoming long wave, it is dubious that there be a strongly deterministic phase relationship between the short wave and the outgoing long wave. This suggests that its contribution to the low-frequency loading is only weakly correlated with the other components.

III – 2 Integration in the DIODORE™ software

The modification of the second order set down is introduced in the DIODORE™ software during the computation of the full QTF matrix.
To take into account the depth variation between the front of the barge and the aft of the barge the computation of the factor $R(\omega_1,\omega_2)\exp[i\alpha((\omega_1,\omega_2)]$ is made by step, as illustrated by the figure3.
To minimize the computation time, an “x” step of 20 m was chosen. It permits to have good agreement between the measurements and the simulation with the three configurations of the barge heading.

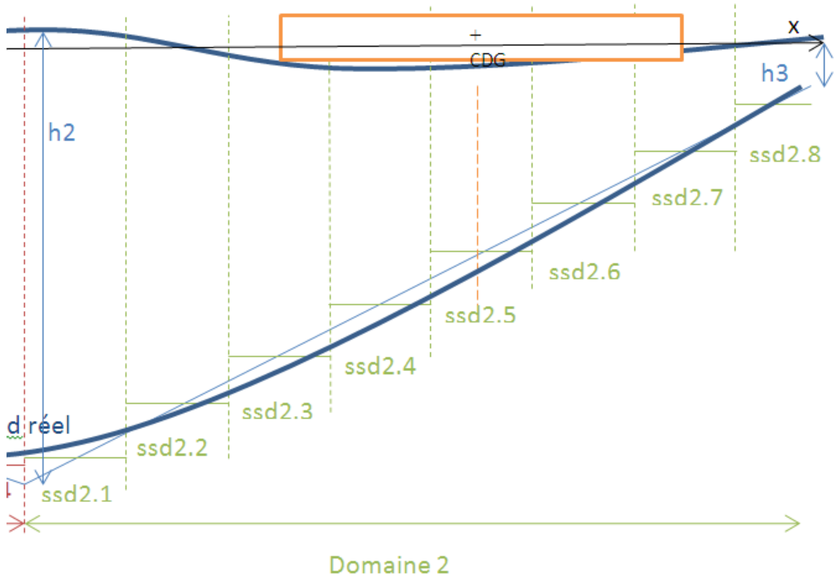

Figure 3 : slope step discretization for the factor R and α computation

$$QTF(\omega_1,\omega_2) = P(\omega_1,\omega_2) + \sum_{x(element\ )} Q_{e(x)}(\omega_1,\omega_2) * R_{e(x)} e^{i\alpha_{e(x)}(\omega_1,\omega_2)} \quad (13)$$

III – 3 Boussinesq model

The so called Boussinesq method was originated by Boussinesq in 1870 [3] for an irrotational and incompressible perfect fluid flow with a free surface, considering that waves propagation in shallow water is a flow with a nearly uniform horizontal velocity from the bottom to the free surface. A particular interest of the method is the reduction of the number of unknowns, since a three dimensional problem can be treated as a two dimensional one and a two dimensional problem can be reduced to a one dimensional problem.
Many improvements have been done since and the latest developments allow the simulation of waves propagation from shallow water to intermediate water depth.
Madsen et al. [9] and Bingham et al [2] developed a Boussinesq type methods which use up to two different orders of interpolation (N=1 and N=2) of the variables along the vertical coordinate. Accordingly differential operators are used to enhance the propagation of waves and the shoaling behaviour. This method was used by [8] (N=1) who found a good agreement between numerical results and semi analytical approach [7]. Guinot [4] used the (N=2) approximation.
The results obtained from development of Guinot [4] are presented below for the case study of the trials run in the BGO First with a sloping bottom. For simulating the propagation of bi-chromatic waves in the BGO First and on the bottom configuration used for the tests of the barge, the simulation strategy is similar to the one adopted by [8]. A first zone located at the 1.05 m water depth is dedicated to the theoretical wave generation, a second zone at the same depth allows for the transition to the numerical model and propagation on a short distance before the sloping bottom. Then the linear 5% sloping bottom extends along the 20 m horizontal distance and finally a damping zone is located at a 0.05 m water depth. The total simulation duration is 12 times the long period associated to the difference frequency of the bi-chromatic waves. Fourier based analyses are then run for a time interval equal to the 8 last long periods. The study is focused on the low frequency behaviour and especially on the evolution of the modulus and phase of the low frequency bounded wave.
The first method used to separate the incoming and outgoing waves is based on a least square analysis of the time signals obtained at different consecutive locations in the tank. An

alternative method uses the spatial derivatives of the signal in the frequency domain. It appears that the first method gives more regular results at the bottom and top of the sloping part of the tank where the slope variation induces some oscillations.
The results on figure 4 shows the comparison of the results obtained from the semi-analytical "step" method and from the Boussinesq model which for the considered couple of period is acceptable. Discrepancy happens with increasing steepness for the abscissa x = 18 to 20 m where the water depth becomes smaller. For higher steepness the water depth at the top of the sloping bottom must be increased to avoid numerical instability.

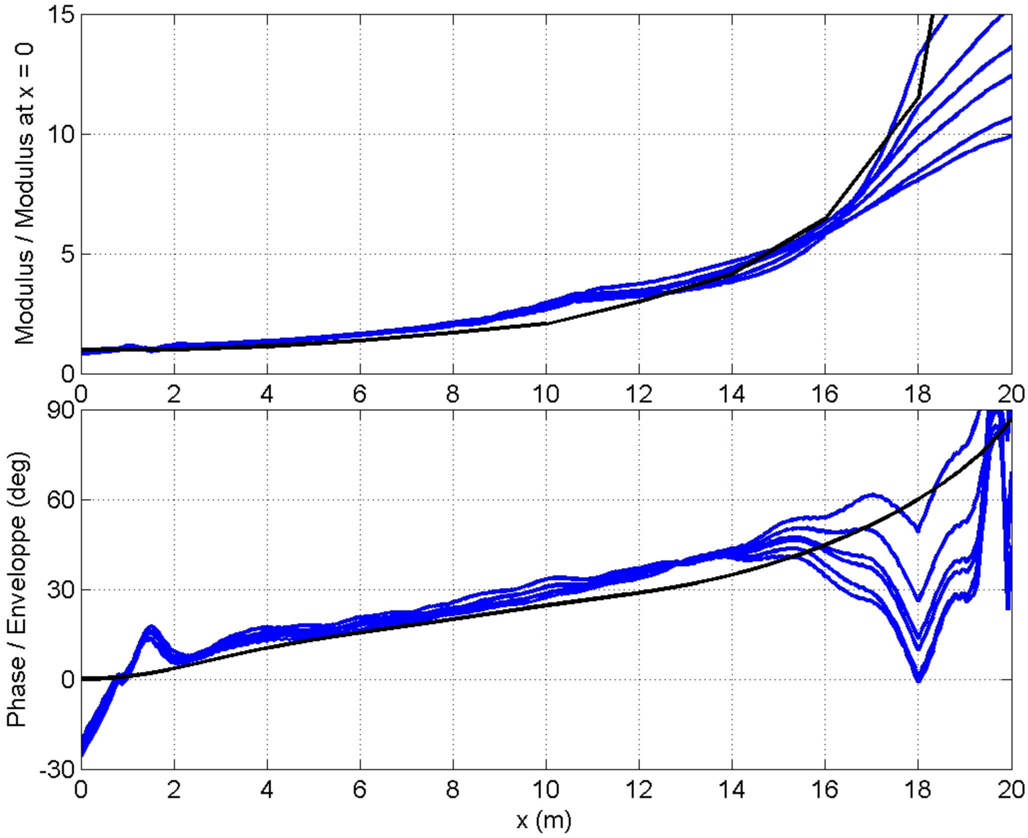

Figure 4 : Bi chromatic waves propagating on the 5% sloping bottom
T = 1.21 s and 1.38 s and various steepness extending from 1 to 4%.
The bounded wave amplitude is divided by its value at the largest depth.
black line : "step" method
blue lines : from Boussinesq method

IV – New calculation of the slow drift motion with DIODORE™

As the cases presented in section III, the Quadratic transfer functions are corrected according to equation (13) and the standard deviation of the dof slow drift motion are recomputed. Results are presented in figure 5 to figure 9 for the different barge locations on the slope.
For the beam seas, as previously shown [8], Newman's approximation over-estimates the measurements even when the set-down contribution is added. When we take into account the shoaling effect on the second order loads, the calculations and the measurements are in good agreement.
For the head seas configurations, the Newman's approximation underestimates the motion whereas the added of the contribution of the shoaling in the set down seems to increase the loading and the obtained motions are in the range of the measurements results

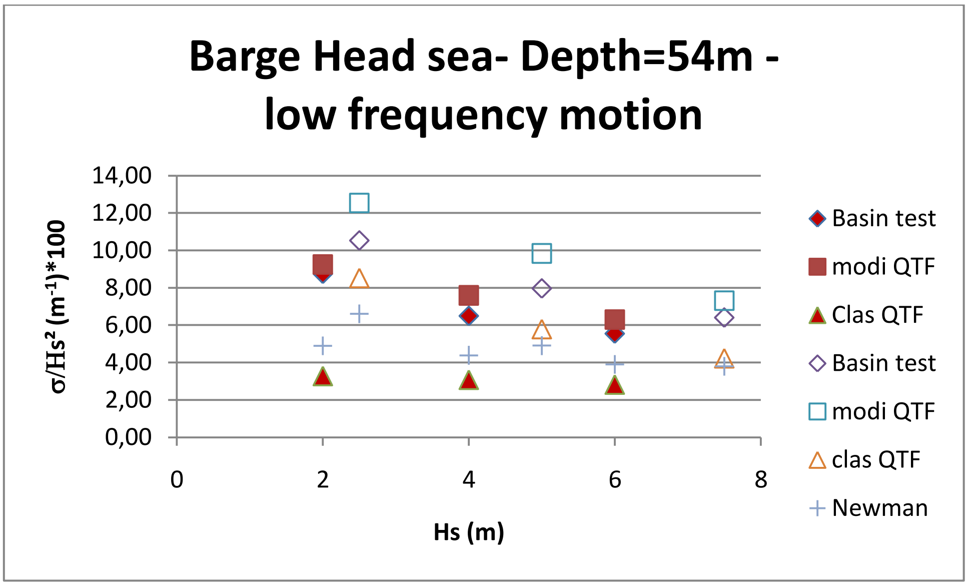

Figure 5 : Head sea results comparisons (depth=54m)

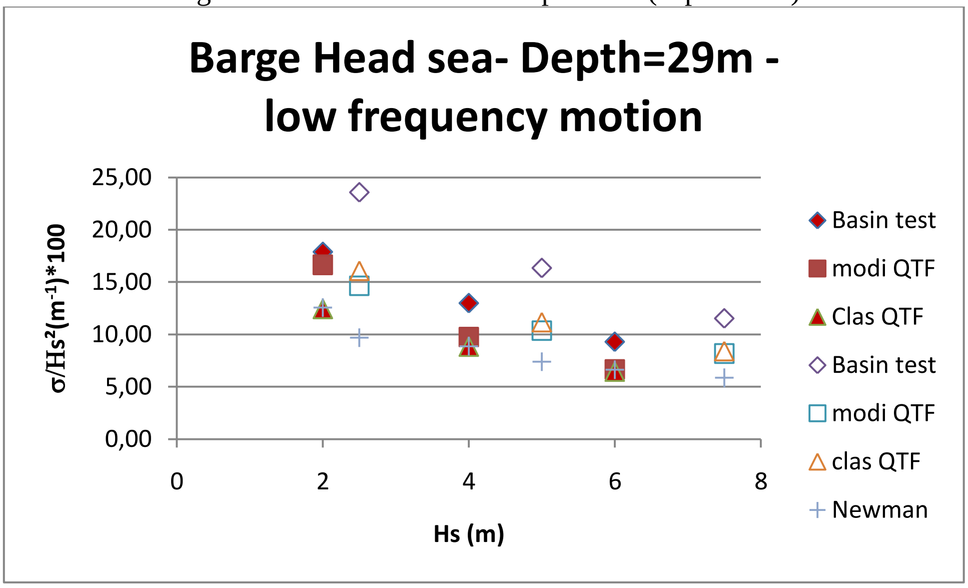

Figure 6 : Head sea results comparisons (depth=29m)

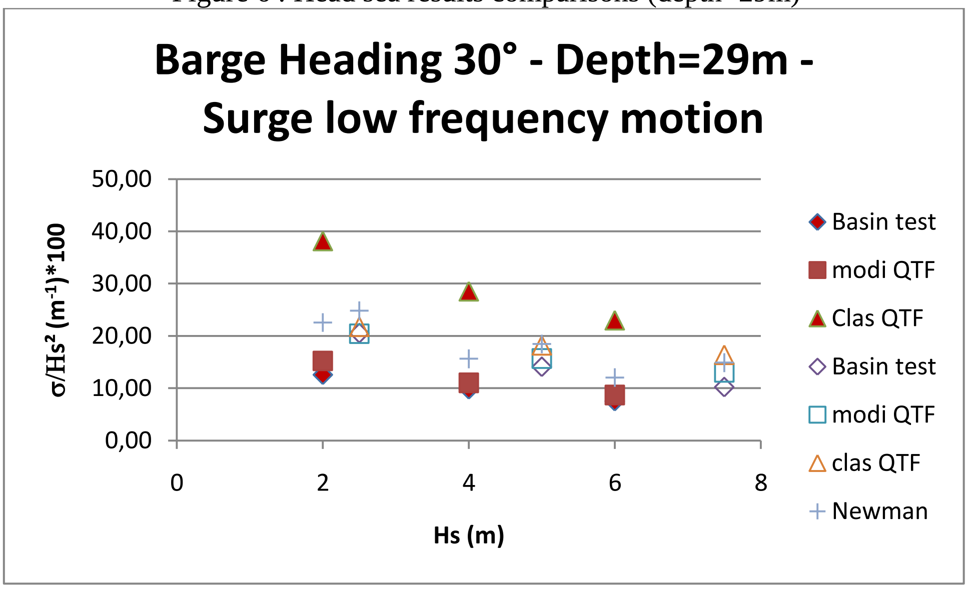

Figure 7 : Heading 30° surge results comparisons (depth=29m)

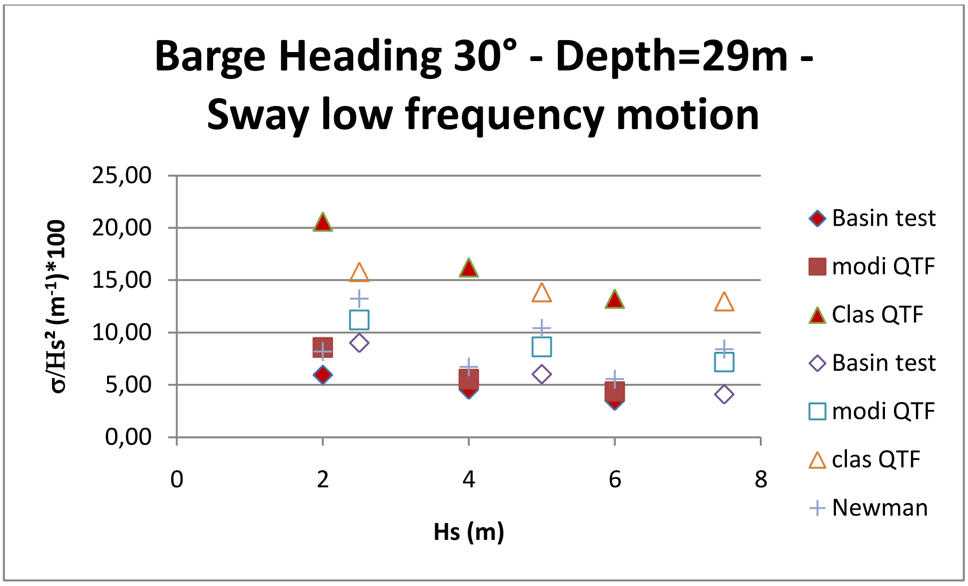

Figure 8 : Heading 30° sway results comparisons (depth=29m)

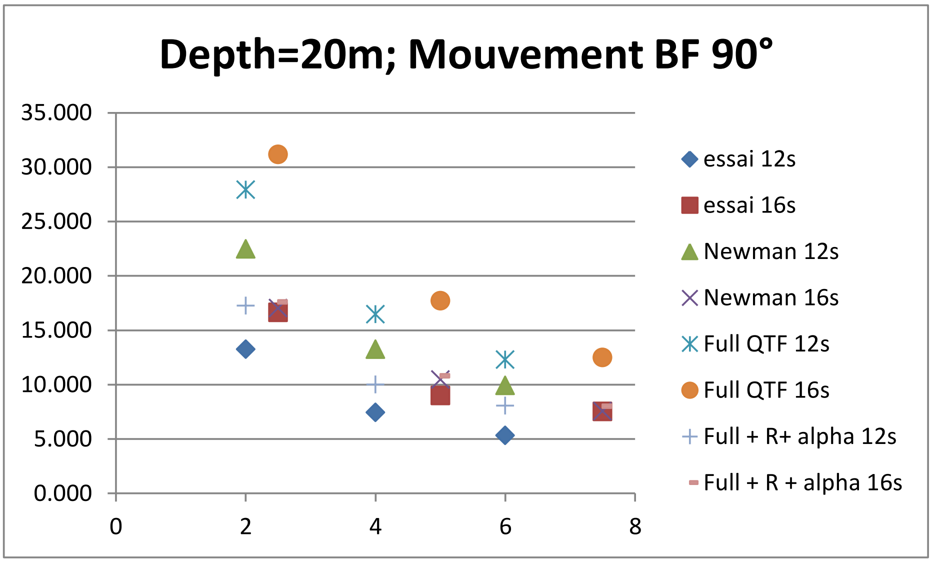

Figure 9 : Beam sea results comparisons (depth=20m)

V – Concluding remarks

A simple numerical model that allows to propagate the second-order long wave, associated with a bi-chromatic wave group, over a variable bathymetry was proposed and integrated in the DIODORE^{TM} software. Even though only unidirectional wave systems, normal to the depth contours, are considered here, the model is applicable to the multidirectional case. The sole restriction is that the bathymetry should be two-dimensional (parallel depth contours). Previously, the model was validated by comparisons with a fully non-linear Boussinesq model.

Experiments were carried out with a rectangular barge moored over the inclined false bottom of the BGO First Basin. Accounting for the amplitude and phase modifications of the long wave contribution to the second order loads, better agreement is obtained between measured and computation slow-drift motion of the barge. To completely validate the method (beam seas, head seas and different heading seas), more comparisons will need to be performed.

Acknowledgement

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Références

[1] J.A. Battjes, H.J. Bakkenes, T.T. Janssen, A.R. Van Dongeren. Shoaling of subharmonic gravity waves. *Journal of Geophysical Research*,109 : C02009, 2004.

[2] H.B. Bingham, P.A. Madsen, and D.R. Fuhrman. Velocity potential formulations of highly accurate Boussinesq-type models. *Coastal Engineering*, 56 : 467–478, 2009.

[3] J. Boussinesq. Théorie des ondes et des remous qui se propagent le long d'un canal rectangulaire horizontal, en communiquant au liquide contenu dans ce canal des vitesses sensiblement pareilles de la surface au fond. *Journal de Mathématique Pures et Appliquées, 2ième Série*, 17 : 55–108, 1872.

[4] F. Guinot. Interactions houle-courant en bathymétrie variable : approches numériques et expérimentales. *PhD thesis, Université de Bretagne Occidentale*, 2010.

[5] A. Ledoux, B. Molin, C. de Jouette, T. Coudray. FPSO roll damping prediction from CFD, 2D and 3D model test investigation. *Proc. 14th Int. Offshore and Polar Eng. Conf. (ISOPE)*, Toulon, France, 2004.

[6] A. Ledoux, B. Molin, G. Delhommeau, F. Remy. A Lagally formulation of the wave drift force. *Proc. 21st Int. Workshop Water Waves & Floating Bodies*, Loughborough (UK), 2006.

[7] Y.N. Liu. Effet d'une bathymétrie variable sur la réponse linéaire et sur le mouvement de dérive lente d'une structure ancrée dans la houle. *PhD thesis, Université de Provence*, 2010.

[8] Y.N. Liu, B. Molin, O. Kimmoun, F. Remy, M-C. Rouault. Experimental and numerical study of the effect of variable bathymetry on the slow drift wave response of floating bodies. *Applied Ocean Research*, 33 : 199-207, 2011.

[9] P. Madsen, D.R. Fuhrman, B. Wang. A Boussinesq-type method for fully nonlinear waves interacting with a rapidly varying bathymetry. *Coastal Engineering*, 53 : 487–504, 2006.

[10] C.C. Mei. The applied dynamics of ocean surface waves. *Wiley-Interscience*, 1983.

[11] B. Molin, F. Remy, Y. Liu. Experimental and numerical study of the slow drift motion of a rectangular barge moored at an inclined beach. *Proc. 29th Int. Conf. on Ocean, Offshore and Arctic Eng. (OMAE2010),* June 6–11, 2010, Shanghai, China. Paper OMAE2010-20623, 2010.

[12] M-C. Rouault, B. Molin, A. Ledoux. A Lagally formulation of the second order slowly varying drift forces. *Proc. 11èmes Journées de l'Hydrodynamique*, 2007.