METHODE DE SINGULARITES MIXTES DE RANKINE ET DE KELVIN POUR LA TENUE A LA MER D'UN NAVIRE ANIME D'UNE VITESSE D'AVANCE DANS LA HOULE

Combined Rankine and Kelvin singularity method for seakeeping of an advancing ship with forward speed in waves

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Résumé

Dans la résolution du problème de la tenue à la mer d'un navire animé d'une vitesse d'avance dans la houle, il est bien connu que la fonction de Green satisfaisant la condition exacte de la surface libre (singularité de Kelvin) présente des propriétaires d'oscillations rapides et de convergence lente rendant l'implémentation numérique extrêmement difficile. La fonction de Rankine est simple et facile à implémenter mais un nombre très important de facettes nécessaire pour satisfaire la condition sur la surface libre rend la méthode difficile à utiliser. Dans le présent travail, nous combinons les deux approches afin de bénéficier la simplicité de la singularité de Rankine dans un domaine près du navire et la satisfaction de condition sur la surface libre de la singularité de Kelvin à une distance du navire. Le couplage des deux approches est réalisé grâce à une surface de contrôle de forme sphéroïdale séparant les deux domaines de fluide.

Summary

The use of the Green function, which satisfies exact free surface boundary, in the seakeeping problem of a ship advancing with non-zero forward speed has some well-known problems such as high oscillation and slow convergence of the wave term. The simple Rankine Green function can be easily implemented, but has some disadvantages such as high number of panels, which represents the free surface and introduction of the damping zone. In this work, these two approaches are combined together in order to benefit the simplicity of the Rankine panel method in the domain near the ship and use the exact forward speed Green function at some distance from the ship. The coupling of tho methods is done with the help of a control surface of the spheroidal form which separates the fluid region onto two domains.

I – Introduction

As it is known, the implementation of the Kelvin source method using the Green function for a moving and oscillating source arises to several well-known problems concerning its wave component. In particular, it behaves with singularities and high oscillations when both the field and source points tend to the free surface [1]. Also, it is the slow convergent integral defined along the dispersion curves. In every cases, for each body shape this wave component should be evaluated very carefully.

In the earlier work [2], numerical results had very good agreement with simple body shapes : a sphere (compared with [3]), a hemisphere [4], an ellipsoid [5] and a Wigley model III [6]. Unfortunately, for the real shape of a container ship, computation was done for the CRS benchmark, the results present large discrepancy from those of model tests.

In this work, we modify the Green function method (Green function satisfies the free surface boundary condition) by combining it with the Rankine panel method.

The Rankine panel method is one of the well-known methods used for sea-keeping problems. It uses the simple Green function $-1/(4\pi r) - 1/(4\pi r^*)$ for unbounded fluid, where r is the distance between the field point and the source point ; r^* is that between the field point and the mirror of the source point with respect to undisturbed free surface. On the other hand, to satisfy the free surface and body boundary conditions, the integrations must be performed over all surfaces. Thus, a large amount of panels are necessary due to panellizing the free surface. In addition, to avoid the reflected wave from the sides of a numerical fluid domain (which is finite) a damping zone has to be introduced.

In this work, we suggest to divide the fluid domain into two subdomains by a control surface of specific shape, semi-spheroid. This surface separates the problem into two problems : 1) the internal one in which the ship is of any form, the Green function is Rankine source Green function, the domain is finite and all normal derivatives of velocity potential Φ are known on the ship hull and on the control surface as the solution of the external problem ; 2) the external one in which the shape of the control surface is known, semi-spheroid, and velocity potential is assumed to be known. Across the control surface two additional conditions must be satisfied : both the velocity potential and its normal derivative are to be continuous. The second problem provides us the Dirichlet-Neumann map which is used to solve the first problem, and as result the original one, by Rankine panel method.

The combination of these two methods keeps their advantages and brings important benefits : area to be discretized becomes smaller, no need to introduce the damping zone, the solution of the problem is that for unbound fluid domain. The calculation of the Green function for ship motion may be done only once for large set of the different ships for the one particular velocity U, incoming wave heading β and frequency ω_0, and an additional parameter which describes the spheroid.

II – Formulation

The reference system moving with the ship at the mean forward speed U along the positive $x-$axis is defined by letting (x, y) plane coincide with the mean free surface and $z-$axis be positive upward, see Figure 1. It is assumed that the fluid is invicid and flow irrotational, the wave steepness is small and the depth is infinite.

The fluid domain is divided into two sub-domains (exterior and interior) by a control surface C which is of known predefined shape, a hemispheroid. The shape of this surface is chosen in such a way in order to use spherical harmonics $S^1_{nm}(\beta, \varphi) = P^m_n(\cos\beta)\sin m\varphi$

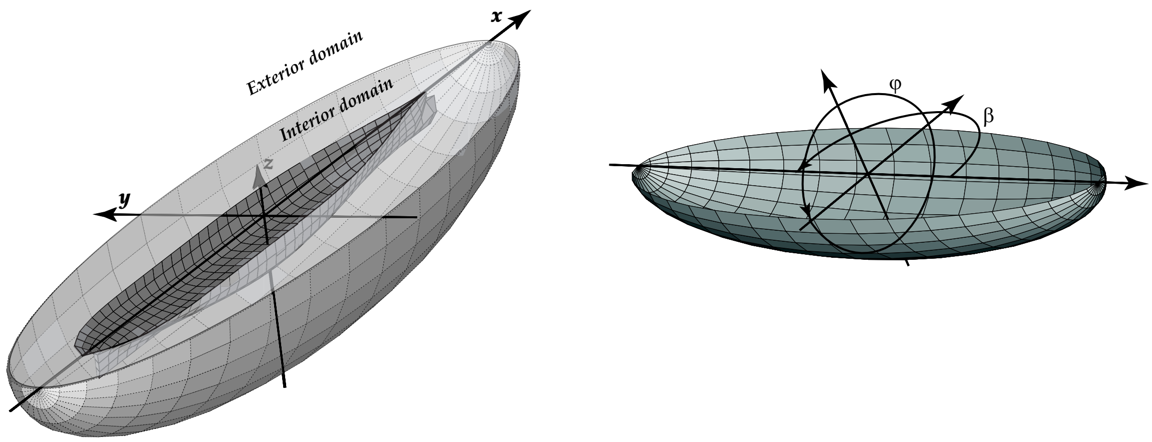

Figure 1 – Formulation of the problem (left) and spheroidal coordinate system on the spheroid (right)

and $S^2_{nm}(\beta, \varphi) = P^m_n(\cos\beta)\cos m\varphi$ to present the velocity potential on C and reduce the side distance from the ship to the control surface. Here $P^m_n(t)$ are the associated Legendre functions. This will allow us to reduce number of free surface panels for the Rankine Panel method, which is used in the interior domain.

From the theory of harmonic functions it is known that any harmonic function $f(x, y, z)$ can be represented in the form of the infinite series with respect to spherical harmonics [7], [8]

$$f(x, y, z) = \sum_{n=0}^{\infty} \frac{f^c_{n0}}{2} S^2_{n0}(\beta, \varphi) + \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left(f^s_{nm} S^1_{nm}(\beta, \varphi) + f^c_{nm} S^2_{nm}(\beta, \varphi), \right) \tag{1}$$

where Cartesian coordinates (x, y, z) and spheroidal coordinates (β, φ) link to each other as the following

$$x = c\cos\beta, \quad y = c\mathcal{R}\sin\beta\cos\varphi, \quad z = c\mathcal{R}\sin\beta\sin\varphi, \tag{2}$$

where $\mathcal{R}$ is the ratio of two radii $\mathcal{R} = R_y/R_x$ and c is a scale factor. The angle $-\pi < \varphi \leqslant 0$, varies in the plane yOz and $0 \leqslant \beta < \pi$ and in the plane inclined to the horizontal plane with angle φ to y-axis, see Fig. 1 right. Note, that equation (1) is valid in either prolate (instead of c we should write $c\cosh\alpha$ and $c\mathcal{R}$ – $c\sinh\alpha$) or oblate (c – $c\sinh\alpha$, $c\mathcal{R}$ – $c\cosh\alpha$) coordinate systems. If we define the coordinate system (2) in such a way that the classical definition of the spherical coordinate system is a particular case when we ought to write

$$x' = c\mathcal{R}\sin\beta\cos\varphi, \quad y' = c\mathcal{R}\sin\beta\sin\varphi, \quad z' = c\cos\beta.$$

But in this coordinate system the axis of the rotation of the ellipse is z, i.e. $R_x = R_y$, while in our case this axis is x-axis, so $R_y = R_z$ and the connection between two spheroidal coordinate systems is $x = z'$, $y = x'$, $z = y'$.

On the control surface we have

$$\begin{aligned}
\phi^{(e)} = \phi^{(i)} &= \sum_{n=0}^{\infty} \frac{\phi^c_{n0}}{2} S^2_{n0}(\beta,\varphi) + \sum_{n=1}^{\infty}\sum_{m=1}^{n} \left(\phi^s_{nm} S^1_{nm}(\beta,\varphi) + \phi^c_{nm} S^2_{nm}(\beta,\varphi)\right);\\
\phi^{(e)}_n = \phi^{(i)}_n &= \sum_{n=0}^{\infty} \frac{\psi^c_{n0}}{2} S^2_{n0}(\beta,\varphi) + \sum_{n=1}^{\infty}\sum_{m=1}^{n} \left(\psi^s_{nm} S^1_{nm}(\beta,\varphi) + \psi^c_{nm} S^2_{nm}(\beta,\varphi)\right),
\end{aligned} \tag{3}$$

where normal vector n is directed inward the exterior domain.

The velocity potential satisfies the Laplace equation in the whole fluid domain :

$$\Delta\phi^{(i)} = 0, \quad \Delta\phi^{(e)} = 0, \tag{4}$$

where 'i' stands for the inner region and 'e' for exterior one. In addition it satisfies the free-surface ($\phi^{(i)}$ and $\phi^{(e)}$) and body ($\phi^{(i)}$) boundary conditions. In order to couple the solutions $\phi^{(i)}$ and $\phi^{(e)}$ in each sub-domain, the conditions of continuity of the velocity potential, $\phi^{(i)} = \phi^{(e)}$, and its normal derivative, $\phi^{(i)}_n = \phi^{(e)}_n$, must be added.

The application of the Green's second identity in the both regions give the following boundary integral equations

$$\int_C \left(\phi^{(e)}_n G - \phi^{(e)} G_n\right) ds = \phi^{(e)}(P), \quad \int_{C+F+B} \left(\phi^{(i)}_n G^R - \phi^{(i)} G^R_n\right) ds = \phi^{(i)}(P), \tag{5}$$

where F means the free surface, B body surface and $G^R = -1/(4\pi r) - 1/(4\pi r^*)$ is the Rankine Green function. Note, that here the integrals are not the principal value integrals and the fluid point is inside the region or on one of its boundary. Otherwise, we have to write the coefficient $1/2$ in front of the ϕ on the right-hand sides of eqs. (5) if the point P belongs to the region's boundary.

Multiplication of the first integral equation in (5) by $1/\pi S^r_{nm}(\beta,\varphi)$ and the following integration of the product yield the linear operator DN which maps the velocity potential by its normal derivatives

$$\vec{\psi} = -DN\vec{\phi}, \quad \psi = (\psi^s_{nm}, \psi^c_{nm})^T, \quad \phi = (\phi^s_{nm}, \phi^c_{nm})^T. \tag{6}$$

The negative sign appears due to the opposite directions of the normal vectors to the control surface for the interior and exterior domains.

After panellization of the free surface, $F = \sum_\alpha F_\alpha$, and body hull, $B = \sum_\beta B_\beta$, the second integral equation in (5) becomes

$$\left(\sum_\alpha \int_{F_\alpha} + \sum_\beta \int_{F_\beta}\right) \left(\phi^{(i)}_n G^R - \phi^{(i)} G^R_n\right) ds + \int_C \left(\phi^{(i)}_n G^R - \phi^{(i)} G^R_n\right) ds = \phi^{(i)}(P), \tag{7}$$

And on the both F_α and B_β we assume the velocity potential and its normal derivative are constant. Thus the above equation (7) can be rewritten as the following

$$\begin{aligned}
\sum_\alpha \psi_\alpha \int_{F_\alpha} G^R ds + \sum_\beta \psi_\beta \int_{F_\beta} G^R ds - \sum_\alpha \phi_\alpha \int_{F_\alpha} G^R_n ds - \sum_\beta \phi_\beta \int_{F_\beta} G^R_n ds&\\
+ \int_C \left(\phi^{(i)}_n G^R - \phi^{(i)} G^R_n\right) ds = \phi^{(i)}(P).&
\end{aligned} \tag{8}$$

Substituting (3) and (6) into (8) gives

$$\sum_{\alpha}\psi_\alpha\int_{F_\alpha}G^R ds+\sum_{\beta}\psi_\beta\int_{F_\beta}G^R ds-\sum_{\alpha}\phi_\alpha\int_{F_\alpha}G_n^R ds-\sum_{\beta}\phi_\beta\int_{F_\beta}G_n^R ds \\ +\mathcal{H}(P)\vec{\phi}+\mathcal{G}(P)DN\vec{\phi}=\phi^{(i)}(P), \tag{9}$$

where the elements of matrices $\mathcal{H}$ and $\mathcal{G}$ are the integrals over C of $G_n S^r_{nm}$ and GS^r_{nm}, respectively.

Depending on the position of the field point P, three cases should be considered : 1) P is on the free surface ; 2) P is on the body hull ; and 3) P is on the control surface C.

In the first two cases, if the point $P = P_\gamma$ locates either on the free surface or body, (9) will be

$$\sum_{\alpha}\psi_\alpha\int_{F_\alpha}G^R ds+\sum_{\beta}\psi_\beta\int_{F_\beta}G^R ds-\sum_{\alpha}\phi_\alpha\int_{F_\alpha}G_n^R ds-\sum_{\beta}\phi_\beta\int_{F_\beta}G_n^R ds \\ +\mathcal{H}(P_\gamma)\vec{\phi}+\mathcal{G}(P_\gamma)DN\vec{\phi}=\phi_\gamma; \tag{10}$$

and if on the control surface, case 3), then we should substitute the velocity potential written on the right-hand side in the form of infinite series (3). The following multiplication by spherical harmonic and the integration of the obtained product provides, after some algebra,

$$\begin{pmatrix}\phi_\alpha\\ \phi_\beta\\ \phi^s_{nm}\\ \phi^c_{nm}\end{pmatrix}=\mathcal{M}\left(\psi_\beta\right), \tag{11}$$

where the vector on the left-hand side $(\phi_\alpha, \phi_\beta, \phi^s_{nm}, \phi^c_{nm})^T$ is unknown, while the vector on the right-hand side (ψ_β) is known from the body boundary conditions.

In (10) the unknown ψ_α are expressed through ϕ_α by applying the free surface boundary condition in order to reach (11).

III – Results

To archive the aim, the work is divided into 3 steps : 1) external problem for the Green function for zero speed in infinitely deep water case ; 2) the Rankine panel method for the internal domain. These two stages together solves the problem for zero speed floating body, which allows us to verify and justify the presented method. 3) external problem for the Green function for an advancing ship, which complete the problem.

At the initial stage, the floating body and the control surface are chosen to be hemispheres of $R = 2$ and $R = 4$ $(R_x = R_y = 4)$, respectively. The choice is done in such a manner, because for the sphere for some parameters (or conditions) there are analytical solutions, which are helpful to validate intermediate results.

For example, if the control surface is the solid body, the normal derivatives of the velocity potential on it are known $\phi_n^{(e)} = n_x$. Thus we may validate the matrix DN : $\vec{\phi}^{(e)} = DN^{-1}\vec{\phi}_n$. Knowing this vector $\vec{\phi}^{(e)}$ we can calculate the velocity potential at any point on the sphere, which is $(1/2)R\phi_n^{(e)}$, [9], in a case when the frequency is zero, $\omega = 0$.

On the other hand, the velocity potential can be computed with the help of the other classical solvers, in our case we used in-house built software Hydrostar.

The comparisons of the results obtained by all these three methods show good agreement (current method for $\omega = 0$ + analytical solution + Hydrostar, current method for $\omega \neq 0$ + Hydrostar).

The next step was the verification of the internal domain solver. The Rankine Green function is corresponding to the problem of a sphere in the unbound fluid with a current (if the body is a hemisphere). Again we had the very good agreement between two different methods.

After confirming that matrices for the both internal and external domains, we performed the coupling.

In Figure 2, the added masses for the surge or sway (left) and heave (right) are shown. The dash line corresponds to the results obtained by in-house built software, while those obtained by the current hybrid method are shown by the solid line.

In Figure 3, there are the damping coefficients for the surge or sway (left) and heave (right). The notations are as same as for the added masses (Fig. 2).

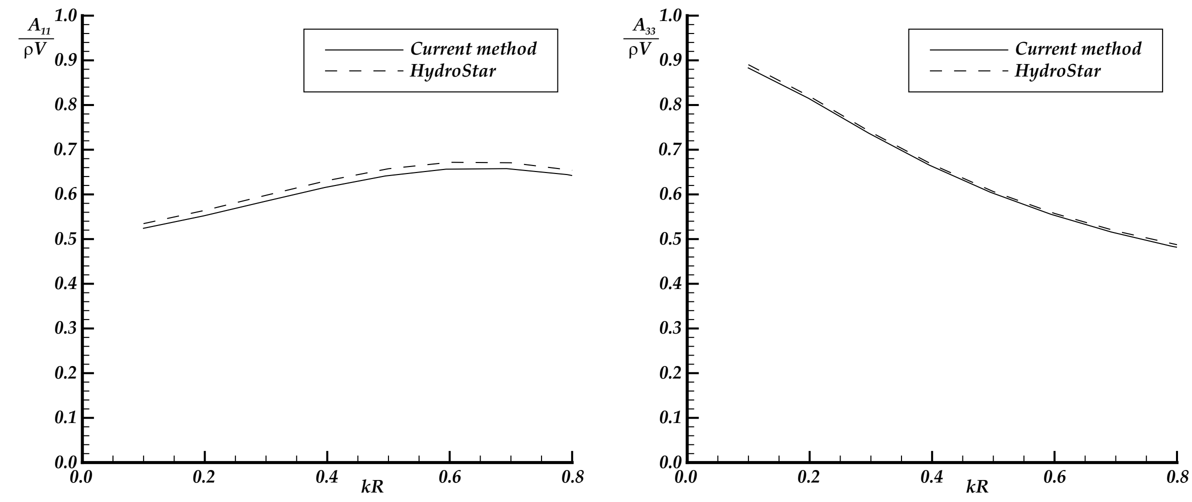

Figure 2 – Added masses for Surge or Sway (left) and Heave motions of the hemisphere of Radius 2

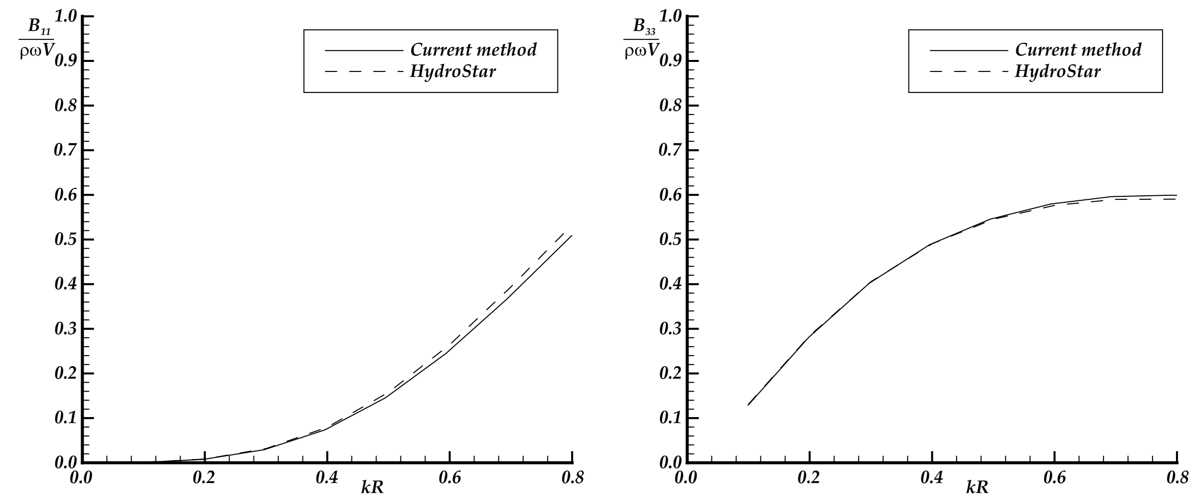

Figure 3 – Damping coefficients for Surge or Sway (left) and Heave motions of the hemisphere of Radius 2

Thus, we can conclude that the present method, at least for the zero-speed Green function, provides rather good results.

IV – Conclusions

In this work we proposed the hybrid method to solve the seakeeping problem for a advancing ship with forward speed in waves. The fluid domain is divided into two sub-domains by introducing a control surface of known shape like hemispheroid. In the inner domain the solution is sought by the Rankine panel method. In order to satisfy the radiation conditions, the solution of the external domain is coupled with Rankine panel method by using the continuity property of the velocity potential and its normal derivative.

The such chosen shape of the control surface provides the following benefits : a) the area to be panellized around the ship is reduced ; b) no need to present the damping zone, because the radiation conditions are satisfied by solving external problem, where the exact forward Green function is used ; c) the solution in the external region is depends only on the ratio of two radii of the spheroid, $\mathcal{R}$ and the frequency ω. Once the exterior problem is solved very accurately, for some set of parameters, the method could be applied to solve seakeeping problems for a wide range of the ships of different geometries more faster than general Rankine panel method and more accurate than the Kelvin source method. d) The integral of the wave component of the exact forward speed Green function when the both source and field points close to each other and to the free surface can be evaluated efficiently.

The validation of this method was demonstrated for the zero-speed Green function for which both the analytical or numerical solutions exist.

V – Future work

In the future, we are going to verify our method by comparing the results calculated by two different methods for other geometries like those given in [3], [4], [5] and [6].

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