A LAGALLY FORMULATION OF THE SECOND-ORDER SLOWLY-VARYING DRIFT FORCES

DÉVELOPPEMENT ET VALIDATION D'UNE FORMULATION LAGALLY DES EFFORTS DU SECOND-ORDRE A BASSE FRÉQUENCE

M.-C. ROUAULT[1], B. MOLIN[2], A. LEDOUX[1]

[1] Principia R.D. (groupe AREVA), Athélia 1, 13705 la Ciotat cedex
[2] Ecole Centrale Marseille, 13451 Marseille cedex 20

Summary

A new formulation of the second-order difference frequency loads is proposed, based on the Lagally theorem. With this formulation, the quadratic part of the QTFs is expressed as the sum of two integrals over the body surface plus one integral over the interior free surface. The good convergence properties of this new formulation are illustrated over three test cases: a vertical cylinder standing on the sea-floor, a floating hemisphere and a FPSO.

Résumé

On présente ici une nouvelle formulation des efforts de deuxième ordre à basse fréquence, basée sur le théorème de Lagally. Dans cette formulation, la partie quadratique des efforts de deuxième ordre s'exprime comme la somme de deux intégrales sur la carène plus une intégrale sur la surface libre intérieure. Des validations sont présentées pour un cylindre vertical posé sur le fond, une sphère flottante et un FPSO.

1 Introduction

This paper is devoted to the numerical evaluation of the so-called QTFs: Quadratic Transfer Functions of the difference frequency wave induced second-order loads. This is an old problem which has been regaining interest in the past years, mostly due to the development of LNG related activity. The reference case is an off-loading terminal, in restricted waterdepth, with the LNG-carrier alongside a GBS or FSRU (Floating Storage Regazeification Unit). To design the mooring system, second-order wave loads acting on either structure are required.

In regular waves the QTFs reduce to the drift force. Most sea-keeping codes compute the drift force following the 'far-field' and/or 'near-field' formulations:

– the far-field formulation was first introduced by Maruo (1964) and extended by Newman (1967) to the drift moment in yaw. In this formulation the change of momentum of the fluid within a control surface surrounding the body is related to the pressure forces acting on the body and on the control surface. The time-averaged value of the hydrodynamic loads can then be related to pressure force and momentum flux at the control surface. When this control surface is taken to infinity the vertical integration can be performed analytically and only one azimuthal integration, involving the Kochin function, needs to be done numerically. This method only yields the horizontal components of the drift force (surge, sway and yaw). When several bodies are involved, it yields the sum of the drift forces, unless several control surfaces are taken surrounding each body. It cannot be extended to computing the QTFs in bichromatic seas.

– the near-field method, proposed by Pinkster and van Oortmerssen (1977), consists in directly integrating the pressure on the body surface, retaining all second-order terms. It yields the six components of the drift force and it can be extended to the calculation of the QTFs in bichromatic seas. Its drawback is that it is less accurate numerically than the far-field method, particularly when the hull has sharp corners where the flow is singular and numerical integration of such terms as $-1/2\,\rho\,\nabla\Phi^2$ presents difficulties.

In Diodore$^{\mathrm{TM}}$, the drift force is obtained following another method, inspired from the Lagally theorem (Landweber 1967). This method was proposed by Guével & Grekas (1981). A more convincing proof of their formulation is given in Ledoux *et al.* (2006). As compared to the two classical formulations, it offers many advantages: it is very easy to implement, it has better convergence properties than the near-field method, it remains valid when several bodies are involved. It has two minor drawbacks: it does not deliver the vertical components of the drift force (except for fully submerged bodies) and usual tricks to remove irregular frequency effects are prohibited.

In irregular seas the full QTF matrix needs to be known to express the slowly-varying drift force. In many cases so-called Newman's approximations, based on the knowledge of the diagonal of the matrix only (the drift force in regular waves), provide an acceptable approximation. When the stiffness of the mooring system increases and/or the waterdepth decreases however, Newman's approximations underestimate the excitation forces, by far. Better approximations, or exact evaluation of the full QTF matrix, are needed.

In this paper we propose a new formulation of the second-order loads in bichromatic seas, in the manner of the Lagally formulation applied to the determination of the drift force, as proposed in Ledoux *et al.* (2006). Section 2 presents the theoretical derivations. Validations are given in the following section.

2 Theoretical developments

2.1 Second-order loads

Be S the wetted hull of the considered body. We denote with a subscript $_0$ the wetted surface S_0 at rest.

We surround the body by a fixed control surface Σ. Be Ω the fluid domain bounded by S, Σ and the free surface within, be F. The normal vector $\overrightarrow{n}$ is outward the fluid domain Ω. The cartesian coordinate system xyz is such that $z = 0$ is the free surface at rest. The waterdepth is constant equal to h.

We rely on potential flow theory, with the velocity potential Φ developed as $\epsilon\,\Phi^{(1)} + \epsilon^2\,\Phi^{(2)} + \ldots$.

Conservation of momentum of the fluid within Ω writes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\iiint_\Omega \rho\,\nabla\Phi\;\mathrm{d}\Omega = -\overrightarrow{F} - \iint_\Sigma p\;\overrightarrow{n}\;\mathrm{d}S - \rho\iint_\Sigma \nabla\Phi\,\frac{\partial\Phi}{\partial n}\;\mathrm{d}S - \iiint_\Omega \rho\,g\,\overrightarrow{k}\;\mathrm{d}\Omega \qquad (1)$$

where $\overrightarrow{F}$ is the hydrodynamic force applied on S and $\overrightarrow{k}$ the vertical unit vector. Equation (1) can be rewritten

$$\overrightarrow{F} = -\rho \frac{\mathrm{d}}{\mathrm{d}t} \iiint_\Omega \nabla\Phi \,\mathrm{d}\Omega - \iint_\Sigma \left[p \,\overrightarrow{n} + \rho \nabla\Phi \frac{\partial\Phi}{\partial n}\right] \mathrm{d}S - \iint_{S\cup\Sigma\cup F} \rho\, g\, z\, \overrightarrow{n} \,\mathrm{d}S \tag{2}$$

Now we take the control surface Σ coinciding with S_0. This means that the volume Ω becomes algebraic, with a local thickness, along S_0, equal to $-\overrightarrow{X}^{(1)} \cdot \overrightarrow{n}_0$ to first-order, $\overrightarrow{X}^{(1)}$ being the local first-order motion of the hull and $\overrightarrow{n}_0$ the normal vector to S_0 into the fluid. Retaining all second-order terms in equation (2), we obtain:

$$\begin{aligned}
\overrightarrow{F}^{(2)} \quad = \quad & \rho \frac{\mathrm{d}}{\mathrm{d}t} \iint_{S_0} \left(\overrightarrow{X}^{(1)} \cdot \overrightarrow{n}_0\right) \nabla\Phi^{(1)} \,\mathrm{d}S + \rho \iint_{S_0} \Phi_t^{(2)} \,\overrightarrow{n}_0 \,\mathrm{d}S \\
& -\frac{1}{2} \rho\, g \int_{\Gamma_0} \eta^{(1)2} \frac{\overrightarrow{n}_0}{\cos\theta} \,\mathrm{d}\Gamma + \rho \iint_{S_0} \left[\frac{1}{2} \left(\nabla\Phi^{(1)}\right)^2 \overrightarrow{n}_0 - \left(\nabla\Phi^{(1)} \cdot \overrightarrow{n}_0\right) \nabla\Phi^{(1)}\right] \mathrm{d}S \\
& + \overrightarrow{F}_{HS}^{(2)} + \rho\, g \int_{\Gamma_0} \frac{\overrightarrow{X}^{(1)} \cdot \overrightarrow{n}_0}{\cos\theta} \,\eta^{(1)} \,\overrightarrow{k} \,\mathrm{d}\Gamma
\end{aligned} \tag{3}$$

Here Γ_0 is the waterline (at rest), $\eta^{(1)}$ is the first-order free surface elevation and θ is the angle between the normal vector $\overrightarrow{n}_0$ and the horizontal plane. This expression agrees with Chen (2007) who started from the pressure integration formulation and applied integral transforms based on Stokes formula (see also Lee, 2006).

The last line in (3) contains only hydrostatic terms acting in the vertical direction. The first term is the hydrostatic restoring force, in still water, taken to second-order (see Molin & Marion 1985). The second one is the weight of water, in between S and S_0, above the still water level. In this paper we will be only concerned with the horizontal components of the QTFs. Hence we discard the last two terms.

Finally we split $\overrightarrow{F}^{(2)}$ into two components, the first one involving quadratic expressions of first-order quantities and the second one involving linearly the second-order potential:

$$\begin{aligned}
\overrightarrow{F}_1^{(2)} \quad = \quad & \rho \frac{\mathrm{d}}{\mathrm{d}t} \iint_S \left(\overrightarrow{X}^{(1)} \cdot \overrightarrow{n}\right) \nabla\Phi^{(1)} \,\mathrm{d}S \\
& -\frac{1}{2} \rho\, g \int_\Gamma \eta^{(1)2} \,\overrightarrow{n}_H \,\mathrm{d}\Gamma + \rho \iint_S \left[\frac{1}{2} \left(\nabla\Phi^{(1)}\right)^2 \overrightarrow{n} - \left(\nabla\Phi^{(1)} \cdot \overrightarrow{n}\right) \nabla\Phi^{(1)}\right] \mathrm{d}S
\end{aligned} \tag{4}$$

$$\overrightarrow{F}_2^{(2)} = \rho \iint_S \Phi_t^{(2)} \,\overrightarrow{n} \,\mathrm{d}S \tag{5}$$

with $\overrightarrow{n}_H$ the normal vector to Γ within the horizontal plane. For the sake of simplicity, since there can no longer be ambiguity, we have dropped the $_0$ subscripts.

2.2 Lagally formulation

Here we are only concerned with the first component, $\overrightarrow{F}_1^{(2)}$.

We assume that the perturbed part of the velocity potential $\Phi^{(1)}$ is generated by a source distribution $\sigma^{(1)}$ on the wetted surface S. As a result a flow is generated inside the hull as well. Hence we distinguish with $_i$ and $_e$ subscripts the velocity potentials inside and outside the body:

$$\Phi_i^{(1)} = \Phi_I^{(1)} + \Phi_{Pi}^{(1)} \qquad\qquad \Phi_e^{(1)} = \Phi_I^{(1)} + \Phi_{Pe}^{(1)} \tag{6}$$

with $\Phi_I^{(1)}$ the incident velocity potential accounting for incoming waves and **perturbations originating from neighboring bodies**.

Within the body, the following identity holds:

$$\iint_{S\cup F_i} \left[\frac{1}{2} \left(\nabla\Phi_i^{(1)}\right)^2 \overrightarrow{n} - \left(\nabla\Phi_i^{(1)} \cdot \overrightarrow{n}\right) \nabla\Phi_i^{(1)}\right] \mathrm{d}S \equiv \overrightarrow{0} \tag{7}$$

with F_i the internal free surface. This well-known identity directly results from

$$\left(\overrightarrow{V}\cdot\nabla\right)\overrightarrow{V}=\frac{1}{2}\,\nabla\left(V^2\right)+\mathrm{Rot}\,\overrightarrow{V}\wedge\overrightarrow{V}$$

Subtracting (7) to (4) and accounting for

$$\nabla\Phi_e^{(1)}-\nabla\Phi_i^{(1)}=\sigma^{(1)}\,\overrightarrow{n} \tag{8}$$

we obtain

$$\begin{aligned}\overrightarrow{F}_1^{(2)} &= \rho\,\frac{\mathrm{d}}{\mathrm{d}t}\iint_S\left(\overrightarrow{X}^{(1)}\cdot\overrightarrow{n}\right)\,\nabla\Phi^{(1)}\,\mathrm{d}S\\ &\quad -\rho\iint_S\sigma^{(1)}\,\left(\nabla\Phi_e^{(1)}-\frac{1}{2}\,\sigma^{(1)}\,\overrightarrow{n}\right)\,\mathrm{d}S-\frac{1}{2}\,\rho\,g\,\int_\Gamma{\eta_e^{(1)}}^2\,\overrightarrow{n}_H\,\mathrm{d}\Gamma+\rho\iint_{F_i}\Phi_{iz}^{(1)}\,\nabla\Phi_i^{(1)}\,\mathrm{d}S\end{aligned}$$

only valid, as written earlier, for the horizontal components.

To progress further and transform the integral over the internal free surface, we must introduce the time dependence of the velocity potential. In Ledoux *et al.* (2006) regular waves are assumed, i.e. one is concerned with the mean drift force (the diagonal of the QTF matrix). Here we consider bichromatic seas.

We assume therefore that the potential $\Phi^{(1)}$ takes the form

$$\Phi^{(1)}(x,y,z,t)=\Re\left\{A_1\,\varphi_1(x,y,z)\,\mathrm{e}^{-\mathrm{i}\,\omega_1 t}+A_2\,\varphi_2(x,y,z)\,\mathrm{e}^{-\mathrm{i}\,\omega_2 t}\right\} \tag{9}$$

with A_1, A_2 the wave amplitudes of the two components at frequencies ω_1 and ω_2. All first-order quantities take a similar form.

We are only interested in second-order quantities at the difference frequency $\omega_1-\omega_2$. They are obtained through the identity

$$\Re\left\{B_1\,\mathrm{e}^{-\mathrm{i}\,\omega_1 t}+B_2\,\mathrm{e}^{-\mathrm{i}\,\omega_2 t}\right\}\times\Re\left\{C_1\,\mathrm{e}^{-\mathrm{i}\,\omega_1 t}+C_2\,\mathrm{e}^{-\mathrm{i}\,\omega_2 t}\right\}=\frac{1}{2}\,\Re\left\{\left(B_1\,C_2^*+B_2^*\,C_1\right)\,\mathrm{e}^{-\mathrm{i}\,(\omega_1-\omega_2)\,t}\right\}+\dots \tag{10}$$

where * means the conjugate complex number.

The difference frequency second-order loads take the form

$$\overrightarrow{F}^{(2)}=\Re\left\{A_1\,A_2\,\overrightarrow{f}^{(2)}\,\mathrm{e}^{-\mathrm{i}\,(\omega_1-\omega_2)t}\right\} \tag{11}$$

with $\overrightarrow{f}^{(2)}(\omega_1,\omega_2)$ twice the QTF as usually defined.

The integral over the interior free surface can then be transformed in the following way

$$\begin{aligned}&\iint_{F_i}\left(\varphi_{1iz}\,\nabla_H\varphi_{2i}^*+\varphi_{2iz}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S=\frac{1}{g}\iint_{F_i}\left(\omega_1^2\,\varphi_{1i}\,\nabla_H\varphi_{2i}^*+\omega_2^2\,\varphi_{2i}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S\\ &=\frac{\omega_1\,\omega_2}{g}\iint_{F_i}\left(\varphi_{1i}\,\nabla_H\varphi_{2i}^*+\varphi_{2i}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S+\frac{\omega_1-\omega_2}{g}\iint_{F_i}\left(\omega_1\,\varphi_{1i}\,\nabla_H\varphi_{2i}^*-\omega_2\,\varphi_{2i}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S\\ &=\frac{\omega_1\,\omega_2}{g}\iint_{F_i}\nabla_H\left(\varphi_{1i}\,\varphi_{2i}^*\right)\,\mathrm{d}S+\frac{\omega_1-\omega_2}{g}\iint_{F_i}\left(\omega_1\,\varphi_{1i}\,\nabla_H\varphi_{2i}^*-\omega_2\,\varphi_{2i}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S\\ &=\frac{\omega_1\,\omega_2}{g}\int_\Gamma\varphi_{1i}\,\varphi_{2i}^*\,\overrightarrow{n}_H\mathrm{d}\Gamma+\frac{\omega_1-\omega_2}{g}\iint_{F_i}\left(\omega_1\,\varphi_{1i}\,\nabla_H\varphi_{2i}^*-\omega_2\,\varphi_{2i}^*\,\nabla_H\varphi_{1i}\right)\,\mathrm{d}S\end{aligned}$$

Here ∇_H designates the horizontal gradient $(\partial/\partial x,\partial/\partial y)$.

On the hull and waterline, the interior and exterior potentials are identical (with a source distribution). As a result the integrals over Γ cancel out. So finally the quadratic second-order force is obtained as

$$\begin{aligned} \vec{f}^{(2)}_1 &= -\frac{1}{2}\,\rho \iint_S (\sigma_1\;\vec{v}^{\,*}_2 + \sigma^*_2\;\vec{v}_1)\;\mathrm{d}S \\ &\quad -\frac{\mathrm{i}\;(\omega_1-\omega_2)}{2}\,\rho \iint_S [(\vec{x}_1\cdot\vec{n})\;\nabla\varphi^*_2 + (\vec{x}^{\,*}_2\cdot\vec{n})\;\nabla\varphi_1]\;\mathrm{d}S \\ &\quad +\frac{\omega_1-\omega_2}{2g}\,\rho \iint_{F_i} (\omega_1\,\varphi_{1i}\;\nabla\varphi^*_{2i} - \omega_2\,\varphi^*_{2i}\;\nabla\varphi_{1i})\;\mathrm{d}S \end{aligned} \tag{12}$$

with

$$\vec{v}_j = \nabla\varphi_j - \frac{1}{2}\,\sigma_j\;\vec{n} = \nabla\varphi_{Ij} + \frac{1}{4\pi}\iint_S \sigma_j(Q)\;\nabla_P G(P,Q)\;\mathrm{d}S_Q,\;\; j=1,2 \tag{13}$$

and G the Green function.

When $\omega_1=\omega_2$ this expression reduces to twice the normalized drift force, as given in Ledoux *et al.* (2006). In that paper it is shown that the Rankine part of the Green function can be removed when computing the velocity $\vec{v}_j$, improving numerical convergence. In bichromatic seas the Rankine part can no longer be removed.

The two additional terms are of order $\omega_1-\omega_2$ and purely imaginary to the leading order (whereas the first term is real to the leading order in $\omega_1-\omega_2$). If only an $O(\omega_1-\omega_2)$ approximation is looked for, they can be obtained as

$$\vec{f}^{(2)}_{1L} = \mathrm{i}\;\rho\,(\omega_1-\omega_2)\;\left[\iint_S \Re\{(\vec{x}\cdot\vec{n})\;\nabla\varphi^*\}\;\mathrm{d}S + \frac{\omega}{g}\iint_{F_i}\Im\{\varphi_i\;\nabla\varphi^*_i\}\;\mathrm{d}S\right] \tag{14}$$

reducing the computational burden (calculations need be done for all ω_i instead of for all couples (ω_i,ω_j)).

It is noteworthy that the Lagally formulation involves an integral over the interior free surface F_i (the calculation of which presents no numerical difficulty).

It is straight-forward to establish that the quadratic moment in yaw, with respect to the mean position G_0 of the center of gravity, is obtained from

$$\begin{aligned} \vec{c}^{(2)}_1 &= -\frac{1}{2}\,\rho \iint_S \vec{r}\wedge(\sigma_1\;\vec{v}^{\,*}_2 + \sigma^*_2\;\vec{v}_1)\;\mathrm{d}S \\ &\quad -\frac{\mathrm{i}\;(\omega_1-\omega_2)}{2}\,\rho \iint_S \vec{r}\wedge[(\vec{x}_1\cdot\vec{n})\;\nabla\varphi^*_2 + (\vec{x}^{\,*}_2\cdot\vec{n})\;\nabla\varphi_1]\;\mathrm{d}S \\ &\quad +\frac{\omega_1-\omega_2}{2g}\,\rho \iint_{F_i} \vec{r}\wedge(\omega_1\,\varphi_{1i}\;\nabla\varphi^*_{2i} - \omega_2\,\varphi^*_{2i}\;\nabla\varphi_{1i})\;\mathrm{d}S \end{aligned} \tag{15}$$

with $\vec{r}$ the vector $(x-x_{G_0}, y-y_{G_0}, z-z_{G_0})$. This is only valid for the yaw moment.

2.3 Contribution due to the second-order potential

In this section we address the evaluation of the second part of the second-order loads, that is

$$\vec{F}^{(2)}_2 = \rho\iint_S \Phi^{(2)}_t\;\vec{n}\;\mathrm{d}S \tag{16}$$

The second-order potential $\Phi^{(2)}$ consists in an incident and a perturbation part:

$$\Phi^{(2)} = \Phi^{(2)}_I + \Phi^{(2)}_D = \Re\left\{A_1\,A_2\;\left[\varphi^{(2)}_I + \varphi^{(2)}_D\right]\;\mathrm{e}^{-\mathrm{i}\,(\omega_1-\omega_2)\,t}\right\} \tag{17}$$

2.3.1 Second-order incident potential

At first-order of approximation the incoming wave system consists in two Airy components, with the free surface elevation

$$\eta^{(1)}_I = A_1\;\sin[k_1\,x\,\cos\beta_1 + k_1\,y\,\sin\beta_1 - \omega_1\,t] + A_2\;\sin[k_2\,x\,\cos\beta_2 + k_2\,y\,\sin\beta_2 - \omega_2\,t] \tag{18}$$

and the velocity potential

$$\Phi_I^{(1)} = -\frac{A_1\,g}{\omega_1}\,\frac{\cosh k_1(z+h)}{\cosh k_1 h}\,\cos[k_1\,x\,\cos\beta_1 + k_1\,y\,\sin\beta_1 - \omega_1\,t] \tag{19}$$

$$-\frac{A_2\,g}{\omega_2}\,\frac{\cosh k_2(z+h)}{\cosh k_2 h}\,\cos[k_2\,x\,\cos\beta_2 + k_1\,y\,\sin\beta_2 - \omega_2\,t] \tag{20}$$

The associated second-order potential taking place at the difference frequency $\omega_1 - \omega_2$ is obtained as (e.g. see Molin, 2002):

$$\varphi_I^{(2)} = -\mathrm{i}\;\frac{q}{-(\omega_1-\omega_2)^2 + g\Delta k\;\mathrm{th}\Delta k h}\;\frac{\mathrm{ch}\Delta k(z+h)}{\mathrm{ch}\Delta k h}\;\mathrm{e}^{\mathrm{i}\,(\vec{k}_1-\vec{k}_2)\cdot\vec{R}} \tag{21}$$

with

$$\vec{k}_1 = \begin{pmatrix} k_1\,\cos\beta_1 \\ k_1\,\sin\beta_1 \end{pmatrix} \qquad \vec{k}_2 = \begin{pmatrix} k_2\,\cos\beta_2 \\ k_2\,\sin\beta_2 \end{pmatrix} \qquad \vec{R} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{22}$$

$$\Delta k = \|\vec{k}_1 - \vec{k}_2\| = \sqrt{k_1^2 + k_2^2 - 2k_1k_2\cos(\beta_1-\beta_2)} \tag{23}$$

$$q = -\frac{1}{2}\left(\frac{\omega_1^3}{\mathrm{sh}^2 k_1 h} - \frac{\omega_2^3}{\mathrm{sh}^2 k_2 h}\right) - \omega_1\omega_2\,(\omega_1-\omega_2)\left(\frac{\cos(\beta_1-\beta_2)}{\mathrm{th}\,k_1 h\;\mathrm{th}\,k_2 h} + 1\right) \tag{24}$$

The contribution of $\varphi_I^{(2)}$ to the loads is easily obtained as

$$\vec{f}_{2I}^{(2)} = -\mathrm{i}\;\rho\,(\omega_1-\omega_2)\iint_S \varphi_I^{(2)}\;\vec{n}\;\mathrm{d}S \tag{25}$$

The associated moment is

$$\vec{c}_{2I}^{(2)} = -\mathrm{i}\;\rho\,(\omega_1-\omega_2)\iint_S \varphi_I^{(2)}\;\vec{r}\wedge\vec{n}\;\mathrm{d}S \tag{26}$$

2.3.2 Second-order diffraction potential

The second-order diffraction potential verifies the Laplace equation in the fluid domain, the no-flow condition at the bottom, decaying conditions at infinity, and the following boundary conditions on the hulls and on the free surface.

Body boundary condition On each hull the second-order diffraction potential $\Phi_D^{(2)}(x,y,z,t)$ verifies the following no-flow condition

$$\nabla\Phi_D^{(2)}\cdot\vec{n} = -\nabla\Phi_I^{(2)}\cdot\vec{n} + \dot{\vec{X}}_1^{(2)}\cdot\vec{n} + \left(\dot{\vec{X}}^{(1)} - \nabla\Phi^{(1)}\right)\cdot\left(\vec{A}^{(1)}\wedge\vec{n}\right) - \left(\vec{X}^{(1)}\nabla\right)\nabla\Phi^{(1)}\cdot\vec{n} \tag{27}$$

or

$$\nabla\Phi_D^{(2)}\cdot\vec{n} = -\nabla\Phi_I^{(2)}\cdot\vec{n} + \dot{\vec{X}}_1^{(2)}\cdot\vec{n} - \left[\vec{A}^{(1)}\wedge\left(\dot{\vec{X}}^{(1)} - \nabla\Phi^{(1)}\right)\right]\cdot\vec{n} - \left(\vec{X}^{(1)}\nabla\right)\nabla\Phi^{(1)}\cdot\vec{n} \tag{28}$$

with $\vec{A}^{(1)}$ the angular motion.

In the right-hand side $\vec{X}_1^{(2)}$ designates the second-order motion that results from the first-order angular motion. It depends on the way the angular motion is defined (Euler angles, etc.). In the end it induces second-order loads that are of order $(\omega_1-\omega_2)^2$, a priori negligible. Hence we discard this term.

Finally, restricting ourselves to components at the difference frequency $\omega_1 - \omega_2$, the body boundary condition for $\varphi_D^{(2)}$ is taken as

$$\begin{aligned}\nabla\varphi_D^{(2)} \cdot \overrightarrow{n} &= -\nabla\varphi_I^{(2)} \cdot \overrightarrow{n} \\ &\quad -\frac{1}{2}\left[\overrightarrow{a}_1 \wedge (\mathrm{i}\,\omega_2\,\overrightarrow{x}_2^* - \nabla\varphi_2^*) + \overrightarrow{a}_2^* \wedge (-\mathrm{i}\,\omega_1\,\overrightarrow{x}_1 - \nabla\varphi_1)\right] \cdot \overrightarrow{n} \\ &\quad -\frac{1}{2}\left[(\overrightarrow{x}_1\,\nabla)\,\nabla\varphi_2^* + (\overrightarrow{x}_2^*\,\nabla)\,\nabla\varphi_1)\right] \cdot \overrightarrow{n} \\ &= -\nabla\varphi_I^{(2)} \cdot \overrightarrow{n} + r^{(2)} \end{aligned} \tag{29}$$

Free surface boundary condition It takes the form:

$$g\,\varphi_{Dz}^{(2)} - (\omega_1 - \omega_2)^2\,\varphi_D^{(2)} = s^{(2)} = s_1^{(2)} + s_2^{(2)} \tag{30}$$

where

$$s_1^{(2)} = \mathrm{i}\,(\omega_1 - \omega_2)\,(\nabla\varphi_1\,\nabla\varphi_2^* - \nabla\varphi_{I1}\,\nabla\varphi_{I2}^*) \tag{31}$$

$$\begin{aligned} s_2^{(2)} &= \frac{1}{2\,g}\left\{-\mathrm{i}\,\omega_1\,\varphi_1\,\left(-\omega_2^2\,\varphi_{2z}^* + g\,\varphi_{2zz}^*\right) + \mathrm{i}\,\omega_2\,\varphi_2^*\,\left(-\omega_1^2\,\varphi_{1z} + g\,\varphi_{1zz}\right)\right\} \\ &\quad -\frac{1}{2\,g}\left\{-\mathrm{i}\,\omega_1\,\varphi_{I1}\,\left(-\omega_2^2\,\varphi_{I2z}^* + g\,\varphi_{I2zz}^*\right) + \mathrm{i}\,\omega_2\,\varphi_{I2}^*\,\left(-\omega_1^2\,\varphi_{I1z} + g\,\varphi_{I1zz}\right)\right\} \end{aligned} \tag{32}$$

Induced loads They take the form:

$$\overrightarrow{f}_{2D}^{(2)} = -\mathrm{i}\,\rho\,(\omega_1 - \omega_2)\iint_S \varphi_D^{(2)}\,\overrightarrow{n}\,\mathrm{d}S \tag{33}$$

This expression is transformed via Haskind's theorem (Molin 1979). For instance we consider the x-component, meaning we want to calculate

$$I_1 = \iint_S \varphi_D^{(2)}\,n_1\,\mathrm{d}S \tag{34}$$

We introduce the linear radiation potential ψ_1 in x, at the frequency $\omega_1 - \omega_2$. If there are several bodies, ψ_1 should satisfy homogeneous Neumann conditions on the neighboring hulls and supplementary integrals over these hulls will result. In the following, for the sake of simplicity, we assume that we have only one body. The integral I_1 is transformed as

$$I_1 = \iint_S \varphi_D^{(2)}\,\nabla\psi_1 \cdot \overrightarrow{n}\,\mathrm{d}S = \iint_S \psi_1\,\nabla\varphi_D^{(2)} \cdot \overrightarrow{n}\,\mathrm{d}S - \iint_{F_e}\left(\psi_1\,\varphi_{Dz}^{(2)} - \varphi_D^{(2)}\,\psi_{1z}\right)\mathrm{d}S \tag{35}$$

$$= \iint_S \psi_1\,\left(-\nabla\varphi_I^{(2)} \cdot \overrightarrow{n} + r^{(2)}\right)\mathrm{d}S + \frac{1}{g}\iint_{F_e} s^{(2)}\,\psi_1\,\mathrm{d}S \tag{36}$$

with F_e the outer free surface.

Therefore $\overrightarrow{f}_{2D}^{(2)}$ splits into three components:

$$\overrightarrow{f}_{2D1}^{(2)} = \mathrm{i}\,\rho\,(\omega_1 - \omega_2)\iint_S \nabla\varphi_I^{(2)} \cdot \overrightarrow{n}\,\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}\mathrm{d}S \tag{37}$$

$$\overrightarrow{f}_{2D2}^{(2)} = -\mathrm{i}\,\rho\,(\omega_1 - \omega_2)\iint_S r^{(2)}\,\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}\mathrm{d}S \tag{38}$$

$$\overrightarrow{f}_{2D3}^{(2)} = -\mathrm{i}\,\rho\,\frac{\omega_1 - \omega_2}{g}\iint_{F_e} s^{(2)}\,\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}\mathrm{d}S \tag{39}$$

and similarly for the yaw moment with ψ_1, ψ_2 replaced with ψ_6.

The calculation of $\overrightarrow{f}_{2D1}^{(2)}$ presents no difficulty. The calculations of $\overrightarrow{f}_{2D2}^{(2)}$ and $\overrightarrow{f}_{2D3}^{(2)}$ are addressed in the following two paragraphs.

Body surface integral The numerical difficulty is mostly associated with the evaluation of the double space derivatives that appear in $r^{(2)}$ (equation (29)). They can be reduced in the manner suggested by Chen (1988) (see also Hung 1988).

Free surface integral We want to compute

$$\overrightarrow{f}^{(2)}_{2D3} = -\mathrm{i}\,\rho\,\frac{\omega_1-\omega_2}{g}\iint_{F_e} s^{(2)}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\mathrm{d}S \tag{40}$$

In the sum frequency case, this free surface integral provides the dominant contribution to the second-order loads and the integration domain must extend very far away from the body (see for instance Molin & Marion 1985). In the difference frequency case, it is often argued that it is negligible or of higher order than $\omega_1-\omega_2$ (Chen 1994). As can be seen in equation (31), $s_1^{(2)}$ is of order $\omega_1-\omega_2$, hence the associated loads are of order $(\omega_1-\omega_2)^2$. As for $s_2^{(2)}$ it is zero identically in deep water if the first-order potentials contain only propagative modes (in $\exp(k_j z)$). In the general case there will be evanescent modes in the immediate vicinity of the hull (except in the case of fixed wall-sided bodies), and after some distance the z dependence of the perturbation parts of φ_1 and φ_2 (be φ_{P1} and φ_{P2}) will be $\cosh k_1(z+h)$ and $\cosh k_2(z+h)$.

The term $s_2^{(2)}$ will then take the form:

$$\begin{aligned}
s_2^{(2)} &= \frac{1}{2\,g}\left\{-\mathrm{i}\,\omega_1\,\varphi_1\,\left(-\frac{\omega_2^4}{g}+g\,k_2^2\right)\,\varphi_2^*+\mathrm{i}\,\omega_2\,\varphi_2^*\,\left(-\frac{\omega_1^4}{g}+g\,k_1^2\right)\,\varphi_1\right\}\\
&\quad-\frac{1}{2\,g}\left\{-\mathrm{i}\,\omega_1\,\varphi_{I1}\,\left(-\frac{\omega_2^4}{g}+g\,k_2^2\right)\,\varphi_{I2}^*+\mathrm{i}\,\omega_2\,\varphi_{I2}^*\,\left(-\frac{\omega_1^4}{g}+g\,k_1^2\right)\,\varphi_{I1}\right\}\\
s_2^{(2)} &= \frac{\mathrm{i}}{2g}\left(\frac{\omega_2\,k_1^2}{\cosh^2 k_1 h}-\frac{\omega_1\,k_2^2}{\cosh^2 k_2 h}\right)\,(\varphi_1\,\varphi_2^*-\varphi_{I1}\,\varphi_{I2}^*)
\end{aligned} \tag{41}$$

meaning that it is of order $\omega_1-\omega_2$ after the evanescent modes of φ_{P1} and φ_{P2} have disappeared.

Here we shall only look for an $O(\omega_1-\omega_2)$ approximation of $\overrightarrow{f}^{(2)}_{2D3}$. This means that the integration domain of the free surface integral can be confined to a small domain in the immediate vicinity of the hull.

Rewriting $s_2^{(2)}$ as

$$\begin{aligned}
s_2^{(2)} &= \frac{1}{2g}\left[-\mathrm{i}\,\omega_1\,\varphi_1\,\left(-\frac{\omega_2^4}{g}\,\varphi_2^*-g\,\varphi_{2xx}^*-g\,\varphi_{2yy}^*\right)+\mathrm{i}\,\omega_2\,\varphi_2^*\,\left(-\frac{\omega_1^4}{g}\,\varphi_1-g\,\varphi_{1xx}-g\,\varphi_{1yy}\right)\right]\\
&\quad-\text{incident x incident}\\
&= \frac{\mathrm{i}}{2g^2}\,\varphi_1\,\varphi_2^*\,\omega_1\,\omega_2\,(\omega_2^3-\omega_1^3)+\frac{\mathrm{i}}{2}\,\left[\omega_1\,\varphi_1\,(\varphi_{2xx}^*+\varphi_{2yy}^*)-\omega_2\,\varphi_2^*\,(\varphi_{1xx}+\varphi_{1yy})\right]\\
&\quad-\text{incident x incident}
\end{aligned} \tag{42}$$

neglecting the first term, of order $\omega_1-\omega_2$, and separating incident and perturbed parts, we retain finally

$$\begin{aligned}
s_2^{(2)} &= \frac{\mathrm{i}}{2}\,\left[k_1^2\,\omega_2\,\varphi_{P2}^*\,\varphi_{I1}-k_2^2\,\omega_1\,\varphi_{P1}\,\varphi_{I2}^*\right]\\
&\quad+\frac{\mathrm{i}}{2}\,\omega_1\,\varphi_1\,\left(\varphi_{P2xx}^*+\varphi_{P2yy}^*\right)-\frac{\mathrm{i}}{2}\,\omega_2\,\varphi_2^*\,\left(\varphi_{P1xx}+\varphi_{P1yy}\right)
\end{aligned} \tag{43}$$

or

$$s_2^{(2)} = -\omega\,\Im\left\{k^2\,\varphi_I\,\varphi_P^*+\varphi\,\left(\varphi_{Pxx}^*+\varphi_{Pyy}^*\right)\right\} \tag{44}$$

since, to our order of approximation, it makes no more sense to distinguish φ_1 and φ_2.

Likewise, since the integration is going to take place in a small neighborhood of the hull, the auxiliary potentials ψ_i can be replaced by their equivalent at zero frequency (double body Kirchhoff potentials) which are real.

The component $\overrightarrow{f}^{(2)}_{2D3}$ is then approximated by

$$f^{(2)}_{2D3i} = \mathrm{i}\,\rho\,\frac{\omega\,(\omega_1-\omega_2)}{g}\,\Im\left\{\iint_{F_e}\left[k^2\,\varphi_I\,\varphi_P^*\,\psi_i - \nabla_H(\varphi\,\psi_i)\,\nabla_H\varphi_P^*\right]\,\mathrm{d}S - \oint_\Gamma \varphi\,\psi_i\,\nabla_H\varphi_P^*\cdot\overrightarrow{n}_H\,\mathrm{d}\Gamma\right\} \quad (45)$$

An advantage of this $O(\omega_1-\omega_2)$ approximation of the free surface integral is that calculations need not be carried out for all couples (ω_i,ω_j), but only for all ω_i. However the domain of validity of our proposed approximation remains to be evaluated over practical cases.

3 Validation

The formulation (12) of the quadratic load $\overrightarrow{f}^{(2)}_1$ has been implemented in the software Diodore$^{\mathrm{TM}}$ of Principia. As for the components due to the second-order potential, they have been partly implemented, that is the components $\overrightarrow{f}^{(2)}_{2I}$ (equation (25)) and $\overrightarrow{f}^{(2)}_{2D1}$ (equation (37)). Still, the main originality of our paper is the formulation of the quadratic load $\overrightarrow{f}^{(2)}_1$ and this is the component that requires validation.

3.1 Vertical cylinder

First we consider the simple case of a vertical cylinder, standing on the sea-floor. The waterdepth h is taken equal to 30 times the radius a. For a fixed structure, when pressure integration is applied, the second-order quadratic force is obtained as

$$\overrightarrow{f}^{(2)}_1 = \frac{1}{2}\,\rho\iint_S \nabla\varphi_1^{(1)}\cdot\nabla\varphi_2^{(1)^*}\,\overrightarrow{n}\,\mathrm{d}S - \frac{\rho}{2\,g}\,\omega_1\,\omega_2\int_\Gamma \varphi_1^{(1)}\,\varphi_2^{(1)^*}\,\overrightarrow{n}\,\mathrm{d}\Gamma \quad (46)$$

This can be evaluated analytically (e.g. see Chen 2001).

According to our Lagally formulation, for a fixed structure, the quadratic force is obtained as

$$\overrightarrow{f}^{(2)}_1 = -\frac{1}{2}\,\rho\iint_S (\sigma_1\,\overrightarrow{v}_2^* + \sigma_2^*\,\overrightarrow{v}_1)\,\mathrm{d}S + \frac{\omega_1-\omega_2}{2g}\,\rho\iint_{F_i}(\omega_1\,\varphi_{1i}\,\nabla\varphi_{2i}^* - \omega_2\,\varphi_{2i}^*\,\nabla\varphi_{1i})\,\mathrm{d}S \quad (47)$$

In the following figures 1 through 3 we present the real and imaginary parts of the QTF (normalized by $\rho\,g\,a$) obtained through (46), as evaluated analytically, and through (47), as implemented in Diodore$^{\mathrm{TM}}$, for three different values of $k_1 a$ (0.2, 0.4 and 0.8), while $(k_1-k_2)/k_1$ varies from 0 to 0.5. It can be checked that the agreement is excellent.

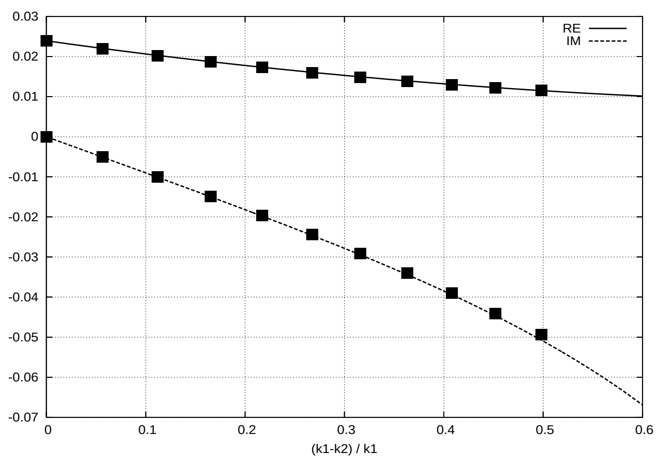

FIG. 1 – *Vertical cylinder. Real and imaginary parts of the normalized QTF in surge, through pressure integration (curves) and Lagally formulation (symbols).* $k_1 a = 0.2$

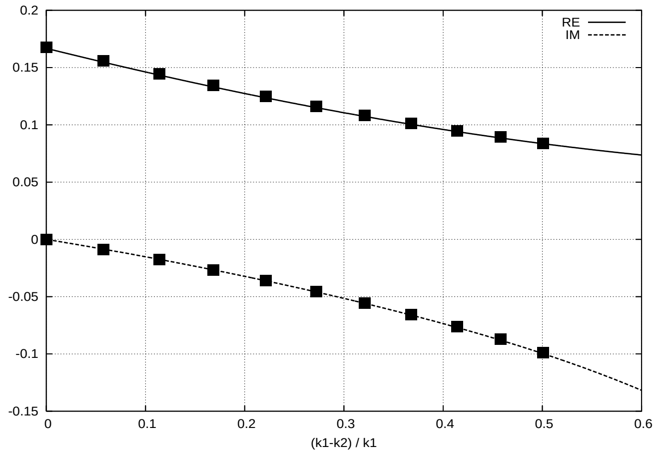

FIG. 2 – *Same as figure 1.* $k_1 a = 0.4$.

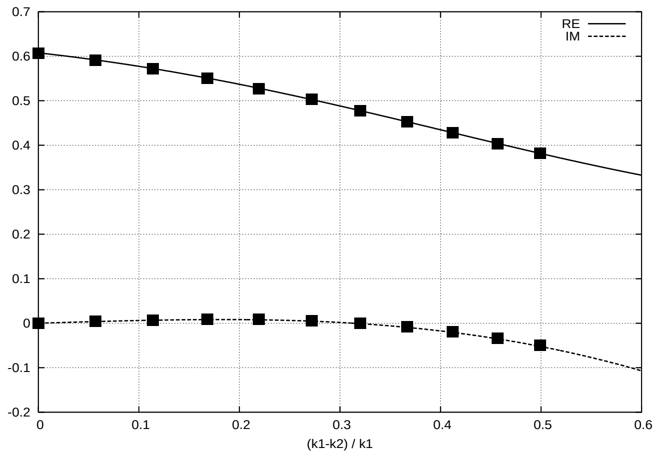

FIG. 3 – *Same as figure 1.* $k_1 a = 0.8$.

3.2 Floating hemisphere

Next we proceed to the case of a floating hemisphere, free to respond to the waves. Reference values for the quadratic part of the second-order horizontal force are taken from Table 7 in Kim & Yue (1990). The waterdepth is equal to 3 times the radius a. Since the sphere is moving a supplementary term appears in our formulation, as compared to the previous case (see equation (12)). Likewise, in the pressure integral method, other terms come into play.

Figure 4 shows the meshes used for the hull and the internal free surface. Then figures 5 through 6 show our numerical values for the normalized QTF, as compared to Kim & Yue's, for different values of $k_2\, a - k_1 a$. As in the cylinder case, only the quadratic part of the QTF is considered here (the first row in Kim & Yue's Table 7). It can be seen that the agreement between both sets of values is quite good.

3.3 FPSO

To conclude we consider the FPSO studied by Lee & Newman (2004) (see also the WAMIT User Manual, 6-21, http://www.wamit.com/manual.html). This FPSO is composed of three portions: "(1) an elliptical bow with a flat horizontal bottom, vertical sides, and semi-elliptical waterlines, (2) a rectangular mid-body with a flat horizontal bottom, vertical sides and constant beam, and (3) a prismatic stern with rectangular sections". The total length is 300 m, the beam 50 m and the draft 25 m. The lengths of the three portions are, respectively, 50, 150 and 100 m, while the reduced beam and draft at the stern are 30 and 15 m.

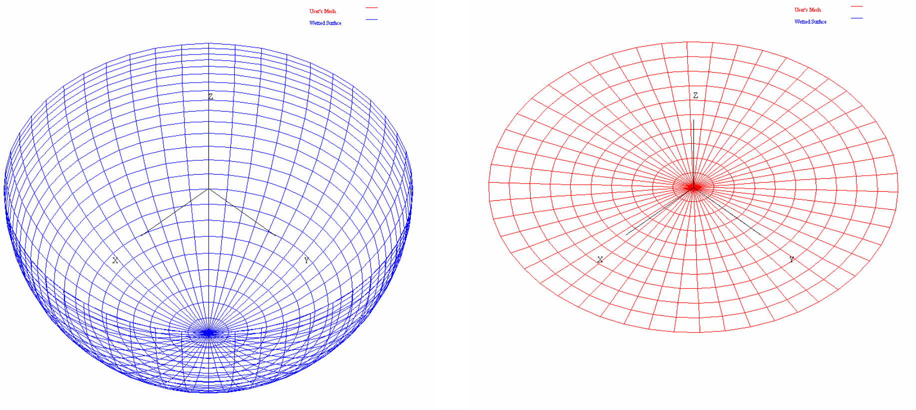

FIG. 4 – *Floating hemisphere. Discretizations of the hull and internal free surface.*

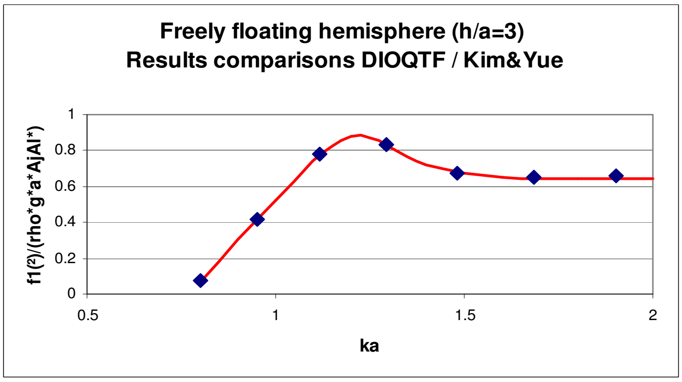

FIG. 5 – *Floating hemisphere. Normalized drift force vs* $k\,a$*. Symbols: Kim & Yue (1990).*

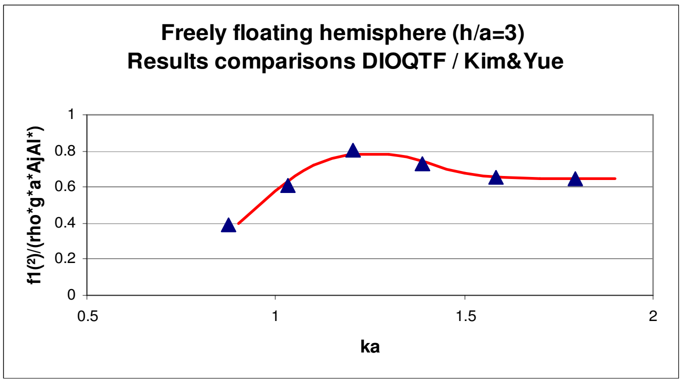

FIG. 6 – *Floating hemisphere. Normalized QTF for* $k_2\,a - k\,a = 0.2$*. Symbols: Kim & Yue (1990).*

The FPSO is considered held fixed in infinite waterdepth and beam waves. The mesh as used in Diodore$^{\text{TM}}$ calculations is shown in figure 8. Figure 9 shows our results, still for the quadratic part of the QTF, as compared to Lee & Newman's (taken from their figure 7-b). Again a good agreement is observed.

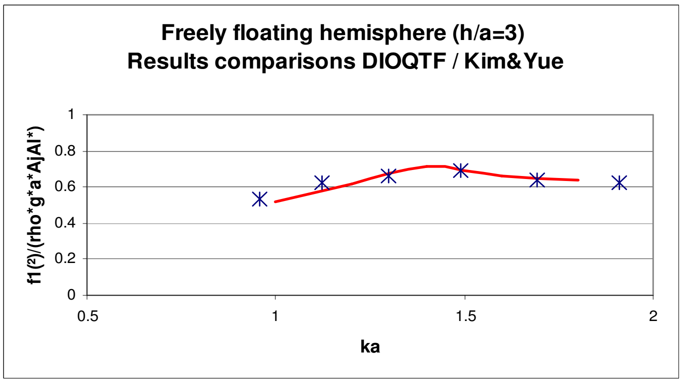

FIG. 7 – *Floating hemisphere. Normalized QTF for* $k_2\,a - k\,a = 0.4$. *Symbols: Kim & Yue (1990).*

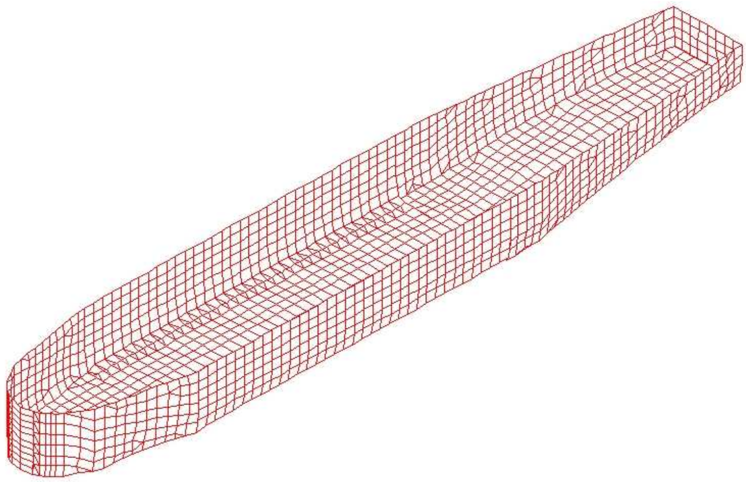

FIG. 8 – *FPSO. Hull mesh.*

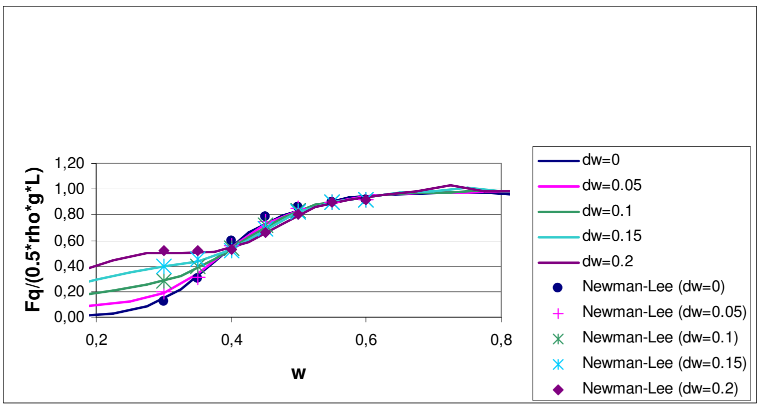

FIG. 9 – *FPSO. Quadratic force in sway.*

4 Concluding remarks

We have proposed a new formulation of the quadratic part of the second-order slowly-varying drift force, based on the Lagally theorem. Its main advantage over the pressure integration method is better convergence properties, for instance when the hull has sharp edges (Ledoux *et al.* 2006). It is easier to implement than the middle-field method proposed by Chen (2007) or Lee (2007). However it is restricted to the horizontal components of the QTFs (except for fully submerged bodies).

We are presently working on the coding of the components $\overrightarrow{f}^{(2)}_{2D2}$ and $\overrightarrow{f}^{(2)}_{2D3}$. Results should be available soon.

Through our formulation it can be seen that, apart from the first term in equation (12) that involves the source densities σ_1 and σ_2, all other terms are of order $\omega_1 - \omega_2$ and purely imaginary in the limit $\omega_2 \to \omega_1$. This feature has also been pointed out and exploited by Chen & Duan (2007).

It suggests the following refinement of the so-called Newman's approximation, based on

$$QTF(\omega_i,\omega_j) \simeq \sqrt{f_d(\omega_i)\; f_d(\omega_j)}\;\mathrm{sign}(f_d) - \mathrm{i}\;(\omega_i - \omega_j)\;\sqrt{\alpha(\omega_i)\;\alpha(\omega_j)}\;\mathrm{sign}(\alpha) \tag{48}$$

with $f_d(\omega_i) = f^{(2)}(\omega_i,\omega_i)/2$ the normalized drift force and $\alpha(\omega_i)$ a real quantity (the sign of which should remain the same when ω_i varies). In Molin & Chen (2002) it is shown that, in the case of a vertical cylinder, the geometric mean provides an excellent approximation of the real part of the QTF. All other terms but the first one in equation (12) can be put in the form i $(\omega_i - \omega_j)\;\alpha(\omega_i)$ for $\omega_j \to \omega_i$ (except, maybe, depending on the waterdepth, the terms $f^{(2)}_{2I}$ and $f^{(2)}_{2D1}$ which involve the second-order incident potential, but which present no computational difficulty).

In irregular waves the second-order slowly varying drift force would then take the form

$$\begin{aligned}
F^{(2)}(t) &= \sum_i \sum_j A_i\, A_j\; \Re\Bigg\{ \Big[\sqrt{f_d(\omega_i)\; f_d(\omega_j)}\;\mathrm{sign}(f_d) \\
&\qquad\qquad -\mathrm{i}\;(\omega_i - \omega_j)\;\sqrt{\alpha(\omega_i)\;\alpha(\omega_j)}\;\mathrm{sign}(\alpha) \Big]\; \mathrm{e}^{-\mathrm{i}\,(\omega_i-\omega_j)t + \mathrm{i}\,(\theta_i - \theta_j)} \Bigg\} \\
&= \sum_i \sum_j A_i\, A_j\; \sqrt{f_d(\omega_i)\; f_d(\omega_j)}\cos\left[(\omega_i - \omega_j)t - \theta_i + \theta_j\right]\;\mathrm{sign}(f_d) \\
&\quad + \frac{\mathrm{d}}{\mathrm{d}t}\left\{ \sum_i \sum_j A_i\, A_j\; \sqrt{\alpha(\omega_i)\;\alpha(\omega_j)}\cos\left[(\omega_i - \omega_j)t - \theta_i + \theta_j\right] \right\}\;\mathrm{sign}(\alpha) \\
&= \left\{ \left[\sum_i A_i\,\sqrt{|f_d(\omega_i)|}\;\cos(\omega_i t - \theta_i) \right]^2 + \left[\sum_i A_i\,\sqrt{|f_d(\omega_i)|}\;\sin(\omega_i t - \theta_i) \right]^2 \right\}\;\mathrm{sign}(f_d) \\
&\quad + \frac{\mathrm{d}}{\mathrm{d}t}\left\{ \left[\sum_i A_i\,\sqrt{|\alpha(\omega_i)|}\;\cos(\omega_i t - \theta_i) \right]^2 + \left[\sum_i A_i\,\sqrt{|\alpha(\omega_i)|}\;\sin(\omega_i t - \theta_i) \right]^2 \right\}\mathrm{sign}(\alpha)
\end{aligned}$$

In this way $F^{(2)}$ is obtained as a combination of single summations squared, in place of a double summation, offering appreciable gains in rapidity. This would hold under the condition that α keeps the same sign when the wave frequency covers the wave spectrum.

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